Rents from Power for a Dissident Elite and Mass Mobilization - Online Appendix

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Abstract

In this appendix, we provide the proofs to the results in the paper and the cases of regime change after 1979. We also discuss the ex-ante likelihood of regime change by introducing a proper prior belief to our benchmark model. Finally we study two variations of the model in the paper (the "model"). The first variation assumes that the mass public is a single decision maker, whose information structure is the same as the information of structure of an individual citizen in the model. The main difference between this variation and the model is that in the former the opposition group faces increased strategic uncertainty. The second variation studies another type of equilibrium in the sequential move game in the paper (the "symbolic leader" case). The symbolic leader is an atomistic citizen who moves first and all other players, including the opposition group, wait until the second period.

1 Proofs

We replicate the equilibrium conditions whenever appropriate for convenience.

The following equations (1), (2), (3) and (4) are the equilibrium conditions for the simultaneous move case.

$$(1-\lambda)F\left(\frac{x^*-\underline{\theta}}{\sigma}\right)=\underline{\theta}. \tag{1}$$

$$\lambda+(1-\lambda)F\left(\frac{x^*-\bar{\theta}}{\sigma}\right)=\bar{\theta}. \tag{2}$$

$$(1+\alpha)G\left(\frac{\bar{\theta}-y^*}{\tau}\right)+(1-\alpha)G\left(\frac{\underline{\theta}-y^*}{\tau}\right)=1-\alpha. \tag{3}$$

$$\frac{1}{\sigma}\int_{-\infty}^{\underline{\theta}} f\left(\frac{\theta-x^*}{\sigma}\right)d\theta+\frac{1}{\sigma}\int_{\underline{\theta}}^{\bar{\theta}} f\left(\frac{\theta-x^*}{\sigma}\right)G\left(\frac{y^*-\theta}{\tau}\right)d\theta=\frac{1+\beta}{2}. \tag{4}$$

Proposition 1 *There exists a unique equilibrium. The equilibrium is in threshold strategies.*

Proof of Proposition 1 This proof closely follows the proof of Proposition 2 in Corsetti et al. (2004). We include here for the sake of completeness.

We will first show that there exists a unique equilibrium in threshold strategies. Establishing that will help us prove the global uniqueness.

It is straightforward to check that equations (1) and (2) have unique solutions in $\underline{\theta}$ and $\bar{\theta}$ respectively given x^*, and that equation (3) has a unique solution in y^* given $\underline{\theta}$ and $\bar{\theta}$. Thus, if equation (4) has a unique solution in x^* as a function of exogenous parameters, the system of equations has a unique solution in terms of the exogenous parameters, which proves the proposition.

Implicitly differentiating (1) and (2) with respect to x^*:

$$\frac{\partial \underline{\theta}}{\partial x^*} = \frac{(1-\lambda) f\left(\frac{x^*-\underline{\theta}}{\sigma}\right)}{\sigma + (1-\lambda) f\left(\frac{x^*-\underline{\theta}}{\sigma}\right)} \in (0,1)$$

$$\frac{\partial \bar{\theta}}{\partial x^*} = \frac{(1-\lambda) f\left(\frac{x^*-\bar{\theta}}{\sigma}\right)}{\sigma + (1-\lambda) f\left(\frac{x^*-\bar{\theta}}{\sigma}\right)} \in (0,1).$$

Implicitly differentiating (3):

$$\frac{1+\alpha}{\tau}\frac{\partial \bar{\theta}}{\partial x^*} g\left(\frac{\bar{\theta}-y^*}{\tau}\right) + \frac{1-\alpha}{\tau}\frac{\partial \underline{\theta}}{\partial x^*} g\left(\frac{\underline{\theta}-y^*}{\tau}\right)$$
$$= \left(\frac{1+\alpha}{\tau} g\left(\frac{\bar{\theta}-y^*}{\tau}\right) + \frac{1-\alpha}{\tau} g\left(\frac{\underline{\theta}-y^*}{\tau}\right)\right)\frac{\partial y^*}{\partial x^*}, \tag{5}$$

which implies that $\frac{\partial y^*}{\partial x^*} > 0$.

Let us adopt the following notation for simplicity

$$z = \frac{\theta - x^*}{\sigma}$$
$$\underline{\delta} = \frac{\underline{\theta} - x^*}{\sigma}$$
$$\bar{\delta} = \frac{\bar{\theta} - x^*}{\sigma}. \tag{6}$$

Then (4) becomes

$$\int_{-\infty}^{\underline{\delta}} \phi(z) dz + \int_{\underline{\delta}}^{\bar{\delta}} f(z) G\left(\frac{y^* - x^* - \sigma z}{\tau}\right) dz = \frac{1+\beta}{2} \tag{7}$$

Note that

$$\frac{\partial \underline{\delta}}{\partial x^*} = \frac{\frac{\partial \underline{\theta}}{\partial x^*} - 1}{\sigma} < 0$$

since $\frac{\partial \underline{\theta}}{\partial x^*} < 1$.

Implicitly differentiation of (1) with respect to x^* implies

$$\frac{\partial \underline{\delta}}{\partial x^*} = -\frac{\frac{\partial \underline{\theta}}{\partial x^*}}{(1-\lambda)f(\underline{\delta})} = -\frac{1}{\sigma+1} < 0.$$

Similarly,

$$\frac{\partial \bar{\delta}}{\partial x^*} = -\frac{1}{\sigma+1} < 0.$$

Note that the signs of $\frac{\partial \underline{\delta}}{\partial x^*}$ and $\frac{\partial \bar{\delta}}{\partial x^*}$ imply that $\frac{\partial y^*}{\partial x^*} - 1 < 0$.

The derivative of (7) with respect to x^* is

$$\frac{\partial \underline{\delta}}{\partial x^*} f(\underline{\delta}) + \frac{\partial \bar{\delta}}{\partial x^*} f(\bar{\delta}) G\left(\frac{y^* - x^* - \sigma\bar{\delta}}{\tau}\right) - \frac{\partial \underline{\delta}}{\partial x^*} f(\underline{\delta}) G\left(\frac{y^* - x^* - \sigma\underline{\delta}}{\tau}\right)$$
$$+ \int_{\underline{\delta}}^{\bar{\delta}} f(z) g\left(\frac{y^* - x^* - \sigma z}{\tau}\right) \frac{1}{\tau}\left(\frac{\partial y^*}{\partial x^*} - 1\right) dz < 0$$

Now, note that

$$\lim_{x^* \to -\infty} \underline{\theta} = 0, \quad \lim_{x^* \to \infty} \underline{\theta} = 1 - \lambda, \quad \lim_{x^* \to -\infty} \bar{\theta} = \lambda \quad \lim_{x^* \to \infty} \bar{\theta} = 1 \Rightarrow$$
$$\lim_{x^* \to -\infty} \underline{\delta} = \lim_{x^* \to -\infty} \bar{\delta} = \infty, \quad \lim_{x^* \to \infty} \underline{\delta} = \lim_{x^* \to \infty} \bar{\delta} = -\infty.$$

Thus, the left-hand side of (7) converges to 1 as x^* diverges to $-\infty$, and converges to 0 as x^* diverges to ∞. By the intermediate-value theorem and implicit function theorem, this proves the proposition.

Global Uniqueness

The argument for global uniqueness is similar to the standard one in the global games.

First, consider the extreme scenario in which an individual believes that no other individual mobilizes against the regime. Then given the signal

$$x = \theta + \sigma\varepsilon,$$

the probability that the regime changes is the posterior probability that $\theta \leq 0$,

$$\frac{1}{\sigma}\int_{-\infty}^{0} f\left(\frac{\theta - x}{\sigma}\right) d\theta,$$

which is strictly decreasing in the signal x. Moreover, as x diverges to $-\infty$, the probability converges to one, and as x diverges to ∞, it converges to zero. Therefore, there exists a unique finite $\underline{x}_0$ such that the payoff from mobilization equals the payoff from refraining. Therefore, for any individual who receives a signal below $\underline{x}_0$ mobilization is a dominant strategy.

Since rationality is common knowledge, no player will expect that an individual who received a signal below $\underline{x}_0$ to refrain from mobilizing. Therefore, every individual and the dissident group will behave accordingly. It is possible to characterize the behavior by thresholds $\underline{\theta}(\underline{x}_0)$, $\bar{\theta}(\underline{x}_0)$ and $y^*(\underline{x}_0)$ $\underline{x}_1$ that are uniquely defined by the indifference conditions in the form of equations (1), (2), (3) and (4) respectively. For convenience we reproduce these equations below:

$$(1-\lambda)F\left(\frac{\underline{x}_0 - \underline{\theta}(\underline{x}_0)}{\sigma}\right) = \underline{\theta}(\underline{x}_0),$$
$$\lambda + (1-\lambda)F\left(\frac{\underline{x}_0 - \bar{\theta}(\underline{x}_0)}{\sigma}\right) = \bar{\theta}(\underline{x}_0),$$
$$(1+\alpha)G\left(\frac{\bar{\theta}(\underline{x}_0) - y^*(\underline{x}_0)}{\tau}\right) + (1-\alpha)G\left(\frac{\underline{\theta}(\underline{x}_0) - y^*(\underline{x}_0)}{\tau}\right) = 1-\alpha$$
$$\frac{1}{\sigma}\left[\int_{-\infty}^{\underline{\theta}(\underline{x}_0)} f\left(\frac{\theta - \underline{x}_1}{\sigma}\right) d\theta + \int_{\underline{\theta}(\underline{x}_0)}^{\bar{\theta}(\underline{x}_0)} f\left(\frac{\theta - \underline{x}_1}{\sigma}\right) G\left(\frac{y^*(\underline{x}_0)}{\tau}\right) d\theta\right] = \frac{1+\beta}{2}.$$

By implicitly differentiating with respect to $\underline{x}_0$, we can show that the last equation above is strictly decreasing in x_1 and increasing in x_0, which implies that $\underline{x}_1 > \underline{x}_0$. Indeed, by iterated elimination of dominated strategies, it is possible to define an increasing sequence of thresholds:

$$\underline{x}_0 < \underline{x}_1 < \underline{x}_2 < \ldots .$$

A symmetric argument shows that there is a decreasing sequence $\{\bar{x}_0, \bar{x}_1, \ldots\}$ of thresholds that characterizes the signals for that refraining is the dominant strategy. Moreover, by definition of these thresholds, we have the following ordering:

$$\underline{x}_0 < \underline{x}_1 < \underline{x}_2 < \ldots < \ldots < \bar{x}_2 < \bar{x}_1 < \bar{x}_0.$$

Since both sequences are monotonic and bounded, both converge to limiting thresholds $\underline{x} \leq \bar{x}$ respectively. Note that the best reply of all individuals and the dissident group to these limiting thresholds $(\underline{\theta}(\underline{x}),\ \bar{\theta}(\underline{x}),\ y^*(\underline{x}),\ \underline{x})$ and $(\underline{\theta}(\bar{x}),\ \bar{\theta}(\bar{x}),\ y^*(\bar{x}),\ \bar{x})$ are solutions to the equations (1), (2), (3) and (4). Since we established in the first step above that this set of equations has a unique solution, $\underline{x} = \bar{x}$. This establishes that the unique equilibrium in threshold strategies is also the unique strategy profile that survives iterative elimination of dominated strategies.

■

Proposition 2 *The equilibrium thresholds, $\bar{\theta}$, $\underline{\theta}$, y^*, and x^* strictly increase in α, the bias in the incentives of the dissident group, and strictly decrease in β, the bias in the incentives of individuals comprising the mass public.*

Proof of Proposition 2 This proof is original to this paper. It is also the main result. Let $b \in \{\alpha, \beta\}$. Implicitly differentiating (1) and (2) with respect to b gives

$$\frac{\partial \underline{\theta}}{\partial b} = \frac{\frac{1-\lambda}{\sigma} f\left(\frac{x^* - \underline{\theta}}{\sigma}\right)}{1 + \frac{1-\lambda}{\sigma} f\left(\frac{x^* - \underline{\theta}}{\sigma}\right)} \frac{\partial x^*}{\partial b} \tag{8}$$

$$\frac{\partial \bar{\theta}}{\partial b} = \frac{\frac{1-\lambda}{\sigma} f\left(\frac{x^* - \bar{\theta}}{\sigma}\right)}{1 + \frac{1-\lambda}{\sigma} f\left(\frac{x^* - \bar{\theta}}{\sigma}\right)} \frac{\partial x^*}{\partial b}. \tag{9}$$

There are two important implications of (8) and (9). First, $\frac{\partial \underline{\theta}}{\partial b}$ and $\frac{\partial \bar{\theta}}{\partial b}$ have the same signs as of $\frac{\partial x^*}{\partial b}$.

Equations (1) and (2) also imply that

$$\frac{\partial \underline{\delta}}{\partial b} = -\frac{\frac{\partial \underline{\theta}}{\partial b}}{(1-\lambda) f(\underline{\delta})} \tag{10}$$

$$\frac{\partial \bar{\delta}}{\partial b} = -\frac{\frac{\partial \bar{\theta}}{\partial b}}{(1-\lambda) f(\bar{\delta})}, \tag{11}$$

which implies that each of $\frac{\partial \underline{\delta}}{\partial b}$ and $\frac{\partial \bar{\delta}}{\partial b}$ has the opposite sign of $\frac{\partial x^*}{\partial b}$.

Implicit differentiation of (3) gives

$$\frac{\partial y^*}{\partial \alpha} = \frac{\frac{1+\alpha}{\tau} g\left(\frac{\bar{\theta} - y^*}{\tau}\right) \frac{\partial \bar{\theta}}{\partial \alpha} + \frac{1-\alpha}{\tau} g\left(\frac{\underline{\theta} - y^*}{\tau}\right) \frac{\partial \underline{\theta}}{\partial \alpha} + 1 + G\left(\frac{\bar{\theta} - y^*}{\tau}\right) - G\left(\frac{\underline{\theta} - y^*}{\tau}\right)}{\frac{1+\alpha}{\tau} g\left(\frac{\bar{\theta} - y^*}{\tau}\right) + \frac{1-\alpha}{\tau} g\left(\frac{\underline{\theta} - y^*}{\tau}\right)}. \tag{12}$$

$$\frac{\partial y^*}{\partial \beta} = \frac{\frac{1+\alpha}{\tau} g\left(\frac{\bar{\theta} - y^*}{\tau}\right) \frac{\partial \bar{\theta}}{\partial \beta} + \frac{1-\alpha}{\tau} g\left(\frac{\underline{\theta} - y^*}{\tau}\right) \frac{\partial \underline{\theta}}{\partial \beta}}{\frac{1+\alpha}{\tau} g\left(\frac{\bar{\theta} - y^*}{\tau}\right) + \frac{1-\alpha}{\tau} g\left(\frac{\underline{\theta} - y^*}{\tau}\right)}. \tag{13}$$

Since $\frac{\partial \underline{\theta}}{\partial \alpha}$ and $\frac{\partial \bar{\theta}}{\partial \alpha}$ both have the same sign, (12) implies that $\frac{\partial y^*}{\partial \alpha}$ is positive if $\frac{\partial x^*}{\partial \alpha}$ is positive and $\frac{\partial y^*}{\partial \beta}$ has the same sign as $\frac{\partial x^*}{\partial \beta}$.

Moreover, if we substitute $\frac{\partial \underline{\theta}}{\partial \alpha}$ and $\frac{\partial \bar{\theta}}{\partial \alpha}$ from equations (8) and (9), we find that

$$\frac{\partial x^*}{\partial \alpha}(\underline{c}\cdot\underline{d}+\bar{c}\cdot\bar{d})+1+\Delta P=(\underline{c}+\bar{c})\frac{\partial y^*}{\partial \alpha},$$
$$\frac{\partial x^*}{\partial \beta}(\underline{c}\cdot\underline{d}+\bar{c}\cdot\bar{d})=(\underline{c}+\bar{c})\frac{\partial y^*}{\partial \beta}.$$

where

$$\underline{c}=\frac{1-\alpha}{\tau}g\left(\frac{\underline{\theta}-y^*}{\tau}\right)$$
$$\bar{c}=\frac{1+\alpha}{\tau}g\left(\frac{\bar{\theta}-y^*}{\tau}\right)$$
$$\underline{d}=\frac{\frac{1-\lambda}{\sigma}f\left(\frac{x^*-\underline{\theta}}{\sigma}\right)}{1+\frac{1-\lambda}{\sigma}f\left(\frac{x^*-\underline{\theta}}{\sigma}\right)}$$
$$\bar{d}=\frac{\frac{1-\lambda}{\sigma}f\left(\frac{x^*-\bar{\theta}}{\sigma}\right)}{1+\frac{1-\lambda}{\sigma}f\left(\frac{x^*-\bar{\theta}}{\sigma}\right)}$$
$$\Delta P=G\left(\frac{\bar{\theta}-y^*}{\tau}\right)-G\left(\frac{\underline{\theta}-y^*}{\tau}\right).$$

Since $\bar{d}$ and $\underline{d}$ are in between 0 and 1,

$$\frac{\underline{c}\cdot\underline{d}+\bar{c}\cdot\bar{d}}{\underline{c}+\bar{c}}\in(0,1).$$

Thus if we assume that $\frac{\partial x^*}{\partial \alpha}$, $\frac{\partial x^*}{\partial \beta}<0$, then

$$\left(\frac{\partial y^*}{\partial \alpha}-\frac{\partial x^*}{\partial \alpha}\right)=\frac{1+\Delta P}{\underline{c}+\bar{c}}+\left(\frac{\underline{c}\cdot\underline{d}+\bar{c}\cdot\bar{d}}{\underline{c}+\bar{c}}-1\right)\frac{\partial x^*}{\partial \alpha}>0, \tag{14}$$
$$\left(\frac{\partial y^*}{\partial \beta}-\frac{\partial x^*}{\partial \beta}\right)=\left(\frac{\underline{c}\cdot\underline{d}+\bar{c}\cdot\bar{d}}{\underline{c}+\bar{c}}-1\right)\frac{\partial x^*}{\partial \beta}>0. \tag{15}$$

On the other hand, $\frac{\partial x^*}{\partial \beta}>0$ implies that $\frac{\partial y^*}{\partial \beta}<\frac{\partial x^*}{\partial \beta}$.

Implicit differentiation of (7) with respect to α gives

$$\frac{\partial \underline{\delta}}{\partial \alpha} f(\underline{\delta}) + \frac{\partial \bar{\delta}}{\partial \alpha} f(\bar{\delta}) G\left(\frac{y^* - x^* - \sigma \bar{\delta}}{\tau}\right) - \frac{\partial \underline{\delta}}{\partial \alpha} f(\underline{\delta}) G\left(\frac{y^* - x^* - \sigma \underline{\delta}}{\tau}\right)$$
$$\int_{\underline{\delta}}^{\bar{\delta}} f(z) \left(\frac{\partial y^*}{\partial \alpha} - \frac{\partial x^*}{\partial \alpha}\right) \frac{1}{\tau} g\left(\frac{y^* - x^* - \sigma z}{\tau}\right) dz = 0, \tag{16}$$

which implies that the term $\left(\frac{\partial y^*}{\partial \alpha} - \frac{\partial x^*}{\partial \alpha}\right)$ has the opposite sign of $\frac{\partial \underline{\delta}}{\partial \alpha}$ and $\frac{\partial \bar{\delta}}{\partial \alpha}$. However, if $\frac{\partial x^*}{\partial \alpha} < 0$, then $\frac{\partial \underline{\delta}}{\partial \alpha}$ and $\frac{\partial \bar{\delta}}{\partial \alpha}$ are positive (from (10) and (11)) and from (14), $\frac{\partial x^*}{\partial \alpha} \leq 0$ also implies that $\left(\frac{\partial y^*}{\partial \alpha} - \frac{\partial x^*}{\partial \alpha}\right)$ is positive, which contradicts (16).

On the other hand, implicit differentiation of (7) with respect to α gives

$$\frac{\partial \underline{\delta}}{\partial \beta} f(\underline{\delta}) + \frac{\partial \bar{\delta}}{\partial \beta} f(\bar{\delta}) G\left(\frac{y^* - x^* - \sigma \bar{\delta}}{\tau}\right) - \frac{\partial \underline{\delta}}{\partial \beta} f(\underline{\delta}) G\left(\frac{y^* - x^* - \sigma \underline{\delta}}{\tau}\right)$$
$$\int_{\underline{\delta}}^{\bar{\delta}} f(z) \left(\frac{\partial y^*}{\partial \beta} - \frac{\partial x^*}{\partial \beta}\right) \frac{1}{\tau} g\left(\frac{y^* - x^* - \sigma z}{\tau}\right) dz = \frac{1}{2}. \tag{17}$$

Equation (17) implies that if $\frac{\partial x^*}{\partial \beta} > 0$,

$$\left(\frac{\partial y^*}{\partial \beta} - \frac{\partial x^*}{\partial \beta}\right) > 0,$$

which is a contradiction since equation (15) implies otherwise.

For any $b \in \{\alpha, \beta\}$, if $\frac{\partial x^*}{\partial b}$ were zero, then $\frac{\partial \underline{\delta}}{\partial b}$ and $\frac{\partial \bar{\delta}}{\partial b}$ would be zero as well. This implies that $\frac{\partial y^*}{\partial \alpha}$ is zero by equation (16). However, equation (14) implies that $\frac{\partial y^*}{\partial \alpha}$ equals to 1, which is again a contradiction.

The equation (15) implies that $\frac{\partial y^*}{\partial \beta}$ is zero; however, equation (17) implies that $\frac{\partial y^*}{\partial \beta} < 0$, which is a contradiction. ■

The following lemma provides a condition for dissident group to be pivotal in the limit as the information precision increases indefinitely.

Lemma 1 *Suppose that σ and $\tau \to 0$. $\lim \bar{\theta} > \lim \underline{\theta}$ if the group size*

$$\lambda \geq \min\left\{\frac{1+\beta+(1+\beta)\alpha}{1+\beta+(5+\beta)\alpha}, \frac{1}{2}\right\}.$$

Proof of Lemma 1 Although this result is related to Proposition 5 in Corsetti et al. (2004), it does not follow from it. The rent parameters unique to our model α and β make the analysis more involved.

The following notation will be helpful throughout the proof. Let

$$\begin{aligned}\bar{\delta} &= \frac{\bar{\theta}-x^*}{\sigma},\\ \underline{\delta} &= \frac{\underline{\theta}-x^*}{\sigma},\\ z &= \frac{\theta-x^*}{\sigma}.\end{aligned} \tag{18}$$

Using this notation, equation (4) becomes

$$\int_{-\infty}^{\underline{\delta}} f(z)dz + \int_{\underline{\delta}}^{\bar{\delta}} f(z)G\left(\frac{y^*-x^*-\sigma z}{\tau}\right)dz = \frac{1+\beta}{2} \tag{19}$$

Suppose that $\lim \bar{\theta} = \lim \underline{\theta}$. Then by equating equations (1) and (2) and using the assumption that the probability density function $f(\cdot)$ is symmetric gives

$$\begin{aligned}(1-\lambda)F(-\underline{\delta}) &= \lambda + (1-\lambda)F(-\bar{\delta}) \Leftrightarrow\\ (1-\lambda)(1-F(\underline{\delta})) &= \lambda + (1-\lambda)(1-F(\bar{\delta})) \Leftrightarrow\\ F(\bar{\delta}) - F(\underline{\delta}) &= \frac{\lambda}{1-\lambda},\end{aligned} \tag{20}$$

where $\underline{\delta}$ and $\bar{\delta}$ are defined in equations (18) as follows:

$$\underline{\delta} = \frac{\underline{\theta} - x^*}{\sigma}, \quad \bar{\delta} = \frac{\bar{\theta} - x^*}{\sigma}.$$

Note that equation (20) requires that $\frac{\lambda}{1-\lambda}$ cannot be greater than 1, or equivalently λ cannot be greater than 1/2.

On the other hand, equation (3) can be written as follows:

$$G\left(\frac{\bar{\theta} - y^*}{\tau}\right) = \frac{1-\alpha}{1+\alpha} G\left(\frac{y^* - \underline{\theta}}{\tau}\right),$$

which implies

$$\begin{aligned} G\left(\frac{\bar{\theta} - y^*}{\tau}\right) &\leq \frac{1-\alpha}{1+\alpha}. \\ \frac{\bar{\theta} - y^*}{\tau} &\leq G^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) \end{aligned} \tag{21}$$

Now, to derive the lower bound on λ consider equation (19), which can be rewritten as follows

$$\int_{-\infty}^{\underline{\delta}} f(z)dz + \int_{\underline{\delta}}^{\bar{\delta}} f(z) G\left(\frac{\sigma}{\tau}\left(\bar{\delta} - z\right) - \frac{\bar{\theta} - y^*}{\tau}\right) dz = \frac{1+\beta}{2}.$$

Then equations (20) and (21) imply that

$$\begin{aligned} F(\underline{\delta}) + G\left(-G^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)\right) \frac{\lambda}{1-\lambda} < \frac{1+\beta}{2} \Leftrightarrow \\ F(\underline{\delta}) < \frac{1+\beta}{2} - \frac{\lambda}{1-\lambda} \frac{2\alpha}{1+\alpha}. \end{aligned}$$

Then since $F(\cdot) \geq 0$,

$$\frac{1+\beta}{2} > \frac{\lambda}{1-\lambda}\frac{2\alpha}{1+\alpha} \Leftrightarrow$$
$$\lambda < \frac{1+\beta+(1+\beta)\alpha}{1+\beta+(5+\beta)\alpha},$$

which establishes the contra-positive of the hypothesis. ■

The following equations are the equilibrium conditions for the Sequential Move case.

$$P(\theta \leq \bar{\theta} | x_M^* \text{ and } y \leq y^*) = \frac{\int_{-\infty}^{\bar{\theta}} f\left(\frac{x_M^*-\theta}{\sigma}\right) G\left(\frac{y^*-\theta}{\tau}\right) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_M^*-\theta}{\sigma}\right) G\left(\frac{y^*-\theta}{\tau}\right) d\theta} = \frac{1+\beta}{2}, \tag{22}$$

$$P(\theta \leq \underline{\theta} | x_R^* \text{ and } y > y^*) = \frac{\int_{-\infty}^{\underline{\theta}} f\left(\frac{x_R^*-\theta}{\sigma}\right) G\left(\frac{\theta-y^*}{\tau}\right) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_R^*-\theta}{\sigma}\right) G\left(\frac{\theta-y^*}{\tau}\right) d\theta} = \frac{1+\beta}{2}. \tag{23}$$

$$(1-\lambda)F\left(\frac{x_R^*-\underline{\theta}}{\sigma}\right) = \underline{\theta} \tag{24}$$

$$\lambda + (1-\lambda)F\left(\frac{x_M^*-\bar{\theta}}{\sigma}\right) = \bar{\theta}. \tag{25}$$

$$(1+\alpha)G\left(\frac{\bar{\theta}-y^*}{\tau}\right) + (1-\alpha)G\left(\frac{\underline{\theta}-y^*}{\tau}\right) = 1-\alpha. \tag{26}$$

The following proposition establishes the existence of an equilibrium in threshold strategies in the sequential case.

Proposition 3 *In the sequential-move game, there exists an equilibrium characterized by the 5-tuble* $(\bar{\theta}, \underline{\theta}, x_M^*, x_R^*, y^*)$*, which satisfies equilibrium conditions* (22) - (26) *if the following condition on signal distribution is satisfied:*

$$\textit{For any } k > 0 \quad \lim_{x \to -\infty} \frac{G(kx)}{F(x)} \geq 1.$$

Proof of Proposition 3 We provide an explicit proof of the existence result stated in Corsetti et al. (2004).

Firstly, examining equations (24), (25) and (26) reveals that the equilibrium thresholds $\bar{\theta}$, $\underline{\theta}$ and y^* have unique and finite solutions given the signal thresholds x_M^* and x_R^*. To establish the existence of solutions to x_M^* and x_R^*, we will introduce the following notation:

$$\begin{aligned} \bar{\delta}_M &= \frac{\bar{\theta} - x_M^*}{\sigma}, \\ z_M &= \frac{\theta - x_M^*}{\sigma}, \\ \bar{\delta}_R &= \frac{\bar{\theta} - x_R^*}{\sigma}, \\ z_R &= \frac{\theta - x_R^*}{\sigma}. \end{aligned} \tag{27}$$

Note that

$$\frac{y^* - x_M^* - \sigma z_M}{\tau} = \frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta} - y^*}{\sigma}\right).$$

Then, equation (22) becomes

$$\frac{\int_{-\infty}^{\bar{\delta}_M} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M}{\int_{-\infty}^{\infty} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M} = \frac{1+\beta}{2}, \tag{28}$$

or equivalently,

$$\frac{1}{1 + \frac{\int_{\bar{\delta}_M}^{\infty} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M}{\int_{-\infty}^{\bar{\delta}_M} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M}} = \frac{1+\beta}{2}. \tag{29}$$

If x_M^* diverges to $-\infty$, $\bar{\delta}_M$ diverges to ∞, and so

$$\frac{\int_{\bar{\delta}_M}^{\infty} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M}{\int_{-\infty}^{\bar{\delta}_M} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M} \to 0, \tag{30}$$

since $\lim \bar{\theta}$ and $\lim y^*$ are finite as x_M^* diverges to $-\infty$.

(30) implies that LHS of equation (29) becomes greater than its RHS.

To show that as x_M^* diverges to ∞, the limit of LHS of equation (28) is less than 1/2, note that

$$\frac{\int_{\bar{\delta}_M}^{\infty} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M}{\int_{-\infty}^{\bar{\delta}_M} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M} \geq \frac{G\left(\frac{\sigma}{\tau}\left(2\bar{\delta}_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right)(1-2F(\bar{\delta}_M))}{F(\bar{\delta}_M)},$$

of which the limit is greater than 1, which sufficient to show that LHS of equation (29) becomes less than its RHS as $x_M^* \to \infty$. ■

Proposition 4 *As $\frac{\sigma}{\tau} \to \infty$, the behavior of the dissident group completely determines individuals' behavior. That is,*

$$\begin{aligned} x_M^* &\to \infty \\ x_R^* &\to -\infty \\ \bar{\theta} &\to 1 \\ \underline{\theta} &\to 0 \end{aligned} \tag{31}$$

Proof of Proposition 4

The proof here closely follows the proof of Proposition 7 in Corsetti et al. (2004). We include the proof for the sake of exposition and completeness.

We will use the notation that we defined in equations (27).

Suppose for a moment that the limit of $\frac{\bar{\theta}-y^*}{\sigma}$ is 0. Then $G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right)$ converges to 1 for any $z < \bar{\delta}_M$, and converges to 0 for any $z > \bar{\delta}_M$. Thus

$$\frac{\int_{\bar{\delta}_M}^{\infty} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M}{\int_{-\infty}^{\bar{\delta}_M} f(z_M) G\left(\frac{\sigma}{\tau}\left(\bar{\delta}_M - z_M + \frac{\bar{\theta}-y^*}{\sigma}\right)\right) dz_M} \to 0.$$

As a result, the LHS of (29), which equals to the LHS of equation (22) converges to 1. Therefore, the viability of mobilizing increases indefinitely for individuals; that is, x_M^* diverges to ∞. A similar argument shows that The LHS of (23) converges then to 0; that is, x_R^* diverges to $-\infty$.

To prove that the limit of $\frac{\bar{\theta}-y^*}{\sigma}$ is actually 0, note equation (26) implies that the limit of $G\left(\frac{\bar{\theta}-y^*}{\tau}\right)$ cannot be 0 or 1. Then

$$\lim \frac{\bar{\theta}-y^*}{\tau} \in (-\infty, \infty).$$

Since

$$\frac{\bar{\theta}-y^*}{\sigma} = \frac{\bar{\theta}-y^*}{\tau}\frac{\tau}{\sigma},$$

$$\lim \frac{\bar{\theta}-y^*}{\sigma} = 0.$$

Finally, if $\tau \to 0$, using an argument similar to the one used in Proposition 4 in the main text, it is straightforward to show that $\lim y^* \neq \lim \bar{\theta}$ leads to contradiction. ■

Proposition 5 *As* $\frac{\sigma}{\tau} \to 0$. *Then*

$$\begin{aligned} \bar{\theta} &\to \lambda + \frac{(1+\beta)(1-\lambda)}{2} \\ \underline{\theta} &\to \frac{(1+\beta)(1-\lambda)}{2}. \end{aligned} \tag{32}$$

Proof of Proposition 5 The analysis here is similar to the one of Proposition 8 in Corsetti et al. (2004).

If $\frac{\sigma}{\tau} \to 0$ equations (22) and (23) converge to

$$\frac{1}{1+\frac{1-F(\bar{\delta})}{F(\bar{\delta})}} = \frac{1+\beta}{2}$$

$$\frac{1}{1+\frac{1-F(\underline{\delta})}{F(\underline{\delta})}} = \frac{1+\beta}{2}.$$

Substituting these limits back to the equations (24) and (25) proves the first part. If the limit of σ is a finite number, then we must have $x_M^* \to \bar{\theta}$ and $x_R^* \to \underline{\theta}$. ■

2 Revolutions between 1979 - 2012

Revolutions between 1979 - 2012

Country	Year	Incumbent	New Ruler	Opposition Group/Party	Year Founded
Chief Executive is Replaced with Opposition Leader					
Iran	1979	Mohammad Reza Pahlavi	Ruhollah Khomeini	Ulama	-
Nicaragua	1979	Anastasio Somoza Debayle	Sandinista NLF	Sandinista NLF	1961
Haiti	1986	Jean-Claude Duvalier	Jean-Bertrand Aristide	Ti Legliz	1974
Philippines	1986	Ferdinand Marcos	Corazon Aquino	United Nationalist Democratic Organization	1980
Bulgaria	1989	Todor Zhikov	Zhelyu Zhelev	Union of Democratic Forces	1989
Czechoslovakia	1989	Gustáv Husák	Václav Havel	Charter 77	1976
Poland	1989	Wojciech Jaruzelski	Lech Wałęsa	Solidarity	1980
Romania	1989	Nicolae Ceauşescu	Ion Iliescu	National Salvation Front	1989
Nepal	1990	Birendra Bir Bikram Shah	Girija Prasad Koirala	Nepali Congress	1947
Zambia	1991	Kenneth D. Kaunda	Frederick Chiluba	Movement for Multi-Party Democracy	1990
Comoros	1997	Mohamed Taki Abdoulkarim (Comoros)	Foundi Abdallah Ibrahim (Anjouan)	Anjouan Liberation Movement	1996

Revolutions between 1979 - 2012 (cont'd)

Country	Year	Incumbent	New Ruler	Opposition Group/Party	Year Founded
Chief Executive is Replaced with Opposition Leader					
Albania	1997	Sali Berisha	Rexhep Meidani	Socialist Party of Albania	1991
Indonesia	1998	Suharto	Abdurrahman Wahid	National Awakening Party	1998
Yugoslavia	1999	Slobodan Milošević (Yugoslavia)	Ibrahim Rugova (Kosovo)	Democratic League of Kosovo	1989
Côte d'Ivorie	2000	Robert Guéï	Laurent Gbagbo	Ivorian Popular Front	1982
Peru	2000	Alberto Fujimori	Alejandro Toledo	Perú Posible	1994
Yugoslavia	2000	Slobodan Milošević	Vojislav Koštunica	Democratic Party of Serbia	1992
Philippines	2001	Joseph Estrada	Gloria Macapagal-Arroyo	Lakas-CMD	1991
Bolivia	2003	Gonzalo Sanchez de Lozada	Eva Morales	Movement for Socialism	1995
Georgia	2003	Eduard Shevardnadze	Mikheil Saakashvil	United National Movement	2001
Liberia	2003	Charles Taylor	Ellen Johnson Sirleaf	Unity Party	1984
Kyrgyzstan	2005	Askar Akayev	Kurmanbek Saliyevich Bakiyev	People's Movement of Kyrgyzstan	2004

Revolutions between 1979 - 2012 (cont'd)

Country	Year	Incumbent	New Ruler	Opposition Group/Party	Year Founded
Chief Executive is Replaced with Opposition Leader					
Ukraine	2005	Viktor Yanukovych	Viktor Yushchenko	Our Ukraine	2001
Nepal	2006	Gyanendra Bir Bikram Shah Dev	Ram Baran Yadav	Nepali Congress	1947
Timor Leste	2006	Mari Alkatiri	José Ramos-Horta	Revolutionary Front for Independent East Timor	1974
Côte d'Ivorie	2007	Charles Konan Banny	Guillaume Soro	Patriotic Movement of Côte d'Ivorie	2002
Thailand	2008	Samak Sundaravej	Abhisit Vejjajiva	Democrat Party	1946
Madagascar	2009	Marc Ravalomana	Andry Rajoelina	Tanora Malagasy Vonona	2007
Iceland	2009	Geir Haarde	Jóhanna Sigurðardóttir	Social Democratic Alliance	2000
Kyrgyzstan	2010	Kurmanbek Saliyevich Bakiyev	Almazbek Atambayev	Social Democratic Party of Kyrgyzstan	1993

Revolutions between 1979 - 2012 (cont'd)

Country	Year	Incumbent	New Ruler	Opposition Group/Party	Year Founded
Chief Executive is Replaced with Opposition Leader					
Egypt	2011	Hosni Mubarak	Mohamed Morsi	Muslim Brotherhood	1928
Libya	2011	Muammar Gaddafi	Mohammed Magariaf	National Front for the Salvation of Libya	1981
Tunisia	2011	Zine El Abidine Ben Ali	Moncef Marzouki	Congress for the Republic	2001
Côte d'Ivorie	2011	Laurent Gbagbo	Alasenna Ouattara	Rally of the Republicans	1994
Maldives	2012	Mohamed Nasheed	Abdulla Yameen	Progressive Party of Maldives	2011
Mali	2012	Amadou Toumani Touré	Ibrahim Boubacar Keïta	Rally for Mali	2001
Romania	2012	Emil Boc	Victor Ponta	Social Democratic Party	1990
Chief Executive is Replaced with Another Ruling Elite					
Ecuador	2005	Lucio Gutiérrez	Rafael Correa	-	-
Yemen	2012	Ali Abdullah Saleh	Abd Rabbuh Mansur Hadi	-	-

Revolutions between 1979 - 2012 (cont'd)

Country	Year	Incumbent	New Ruler	Opposition Group/Party	Year Founded
Chief Executive is Replaced with a Neutral Leader					
Guinea	2007	Eugène Camara	Lansana Kouyaté	United Trade Union Guinean Workers	1995
Chief Executive is Overthrown without Replacement					
German DR	1989	Erich Honecker	-	-	-
Somalia	1991	Siad Barre	-	-	-

Note: There are 1779 collective actions between 1979–2012 that are recorded in Banks et al. data set. We found 42 instances of collective actions that triggered or caused the removal of an incumbent Chief Executive (Prime Minister, President, General Secretary of Central Committee or King). There are 2 cases, where the overthrown incumbent is not replaced with a new leader. There are 2 cases where the new leader was a high-rank executive who served before regime change under the overthrown incumbent. There is only one case, where the new leader was neither part of governing elite nor the opposition. In the remaining 37 cases, an incumbent Chief Executive is replaced with a new leader from opposition.

3 Ex-ante Probability of Revolution

So far we have not specified the nature of θ. We have assumed throughout the paper that information about θ is only revealed through the private signals players receive. Players' common prior is, as a result, the improper uniform distribution over the real line. It might be reasonable, however, to assume that individuals share a common belief about the strength of the regime prior to what they learn immediately before or during a rebellion. This assumption can be easily embedded in our model by specifying a prior distribution of θ. Equilibrium beliefs in this case constitute a joint distribution determined by the prior distribution of θ and the distributions of the noise of the signals.

Suppose that θ is determined according to a symmetric distribution over $\mathbb{R}$. Let the mean value of θ be θ_0, its cumulative distribution function $H(\cdot)$, and its density $h(\cdot)$.

The posterior belief about θ given each individual's signal is presented by the following density function:

$$\frac{f\left(\frac{x_i-\theta}{\sigma}\right)h(\theta-\theta_0)}{\int_{-\infty}^{\infty} f\left(\frac{x_i-\theta}{\sigma}\right)h(\theta-\theta_0)d\theta}.$$

Using this expression, we can re-do our analysis. For the simultaneous-move case, the equilibrium thresholds x^*, y^*, $\bar{\theta}$ and $\underline{\theta}$ are the solutions to the following system of equations:

$$\begin{aligned}
(1-\lambda)F\left(\frac{x^*-\underline{\theta}}{\sigma}\right) &= \underline{\theta} \\
\lambda+(1-\lambda)F\left(\frac{x^*-\bar{\theta}}{\sigma}\right) &= \bar{\theta} \\
\frac{(1+\alpha)\int_{-\infty}^{\bar{\theta}} g\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta+(1-\alpha)\int_{-\infty}^{\underline{\theta}} g\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta}{\int_{-\infty}^{\infty} g\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta} &= 1-\alpha \\
\frac{\int_{-\infty}^{\underline{\theta}} f\left(\frac{\theta-x^*}{\sigma}\right)h(\theta-\theta_0)d\theta+\int_{-\underline{\theta}}^{\bar{\theta}} f\left(\frac{\theta-x^*}{\sigma}\right)G\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta}{\int_{-\infty}^{\infty} f\left(\frac{\theta-x^*}{\sigma}\right)h(\theta-\theta_0)d\theta} &= \frac{1+\beta}{2}
\end{aligned} \tag{33}$$

For the sequential-move case, the equilibrium thresholds x_M^*, x_R^*, y^*, $\bar{\theta}$ and $\underline{\theta}$ are determined from:

$$
\begin{aligned}
(1-\lambda)F\left(\frac{x_R^*-\underline{\theta}}{\sigma}\right) &= \underline{\theta} \\
\lambda+(1-\lambda)F\left(\frac{x_M^*-\bar{\theta}}{\sigma}\right) &= \bar{\theta} \\
\frac{(1+\alpha)\int_{-\infty}^{\bar{\theta}} g\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta+(1-\alpha)\int_{-\infty}^{\underline{\theta}} g\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta}{\int_{-\infty}^{\infty} g\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta} &= 1-\alpha \\
\frac{\int_{-\infty}^{\bar{\theta}} f\left(\frac{x_M^*-\theta}{\sigma}\right)G\left(\frac{y^*-\theta}{\tau}\right)h(\theta-\theta_0)d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_M^*-\theta}{\sigma}\right)G\left(\frac{y^*-\theta}{\tau}\right)h(\theta-\theta_0)d\theta} &= \frac{1+\beta}{2}, \\
\frac{\int_{-\infty}^{\underline{\theta}} f\left(\frac{x_R^*-\theta}{\sigma}\right)G\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_R^*-\theta}{\sigma}\right)G\left(\frac{\theta-y^*}{\tau}\right)h(\theta-\theta_0)d\theta} &= \frac{1+\beta}{2}. \qquad (34)
\end{aligned}
$$

These equations, however, do not provide closed-form solutions, so we solve them numerically. Figure 1 illustrates the difference between the impact of the rents of the leader, α, on the ex-ante probability of revolution:

Figure 1 highlights the nuanced effect of rents from power to the dissident group. As we noted above, higher rents are associated with a more aggressive mass public, and therefore a higher ex-ante probability of revolution. On the other hand, for the sequential case, the decision of the dissident group is observed so, if anything, rents make the dissident group less reliable. Higher rents are associated with more cautious individuals therefore lowering the probability of revolution.

Another implication indicated by Figure 1 is that the probability of revolution is always higher in the sequential case. This is because, the informational power of the dissident group is higher in the sequential case. The dissident group is more capable of leading the mass public towards its favorite outcome, regime change.

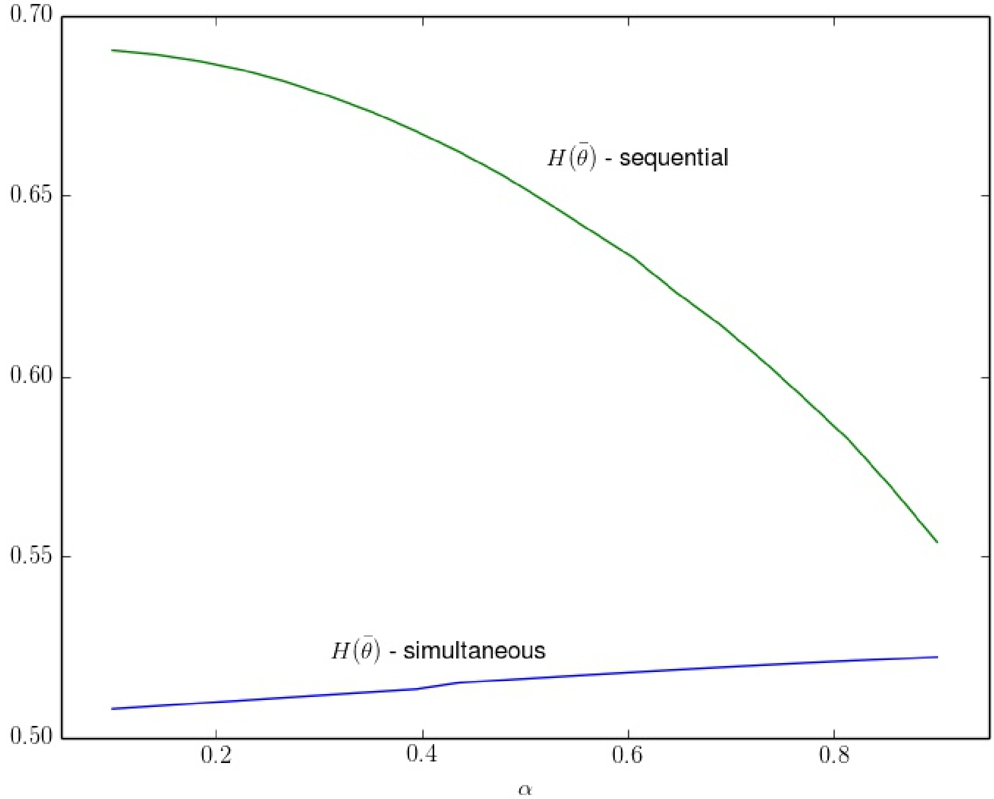

Figure 1: Ex-ante Probability of Revolution as Rents to the Leader Increases

$\sigma = 1$, $\tau = 0.5$, $\beta = 0.1$ $\lambda = 0.1$.

4 Mass Public as a Single Player

In the benchmark model in the main body of the paper (hereafter, we will use "benchmark model" or just "model" to refer to main model in the paper), each citizen interacts simultaneously with each other and with the opposition elite. In the model, each citizen's mass in the population is atomistic, so they do not consider the impact of their decision on the overall level of mobilization. Moreover, the fact that there is an uncountable number of citizens eliminates the strategic uncertainty the opposition elite faces. When we assume the mass public is a single strategic player, in contrast, it has to consider its pivotal role in regime change. Although this model may seem less realistic than the one in which the mass public comprises many individuals, it is worth exploring as it exposes the consequences of two players being pivotal. This is the fundamental difference between the model and the variation considered here. We use capital initials to refer to Mass Public when we refer to the single-player variation.

Precisely, assume that the Mass Public receives a private signal with the same information content as one citizen would have received in the benchmark model. That is, the Mass Public receives

$$x = \theta + \sigma\varepsilon,$$

and the opposition elite receives

$$y = \theta + \tau\eta,$$

where ε and η are independent random variables, whose distributions are symmetric around zero.

Both the opposition elite and the Mass Public simultaneously choose whether to mobilize against the regime conditional on their private information about the strength of the regime θ. The size of the opposition elite is λ and the size of the Mass Public is $1-\lambda$. Let s_{OE} (resp. s_{MP}) denote the mobilization decision of the opposition elite (resp. Mass Public). The incidence of mobilization z is then

$$z = s_{OE}\lambda + s_{MP}(1-\lambda).$$

The regime changes if $z \geq \theta$. We assume without loss of generality that the mobilization size λ of the opposition elites is less than 1/2. The analysis with $\lambda \geq 1/2$ is analogous.

As in the benchmark model, we concentrate on the equilibrium in threshold strategies. We show there exist thresholds (x^*, y^*) such that the opposition elite (resp. Mass Public) chooses to mobilize if and only if its private signal $x \leq x^*$ (resp. $y \leq y^*$).

The strategic interaction between the Mass Public and the opposition elite here is different from the one in the benchmark model. As it is mentioned above, in the model an individual citizen cannot be pivotal. Thus, each citizen decides to mobilize based solely upon the trade-off between participating in an unsuccessful protest and missing on the opportunity to become a part of regime change. Moreover, since there is a continuum of citizens in the model, individual differences between citizens cancel out at the aggregate level, which eliminates the strategic uncertainty the elite face about the response of the mass public. Each player knows exactly the ratio of citizens who will choose to mobilize against the regime given any strength of the regime, θ. The only remaining strategic uncertainty is about the response of the opposition elite.

When the Mass Public is a single player, however, it is no longer the case that the Mass Public's response is deterministic. The response of the Mass Public depends on the particular realization of its private signal, therefore it is impossible for the elite to infer the behavior of the Mass Public with certainty for any given regime strength θ. This feature causes an additional non-linearity in the posterior assessments of players regarding regime change. We describe the posterior beliefs of players below.

We first consider the actions of the opposition elite. The elite mobilizes a λ share of the population, if it decides to do so. Therefore if $\theta \leq \lambda$, the regime changes irrespective of the action of the Mass Public. If θ is between λ and 1, the regime changes only if both players mobilize. The Mass Public mobilizes if $x \leq x^*$. The expected payoff to the elite for mobilizing is

$$(1 + \alpha)(P(\theta \leq \lambda|y) + P(\lambda < \theta \leq 1 \quad \& \quad x \leq x^*|y)).$$

Similarly, the payoff from refraining is

$$(1 - \alpha)(P(\theta > 1 - \lambda|y) + P(0 < \theta \leq 1 - \lambda \quad \& \quad x > x^*|y)).$$

The elite's posterior belief about θ is:

$$\begin{aligned} P(\theta \leq \lambda | y) &= \int_{-\infty}^{\lambda} P(\theta|y) d\theta = \frac{\int_{-\infty}^{\lambda} P(y|\theta) h(\theta - \theta_0) d\theta}{\int_{\infty}^{\infty} P(y|\theta) h(\theta - \theta_0) d\theta} \\ &= \frac{\int_{-\infty}^{\lambda} g\left(\frac{y-\theta}{\tau}\right) h(\theta - \theta_0) d\theta}{\int_{\infty}^{\infty} g\left(\frac{y-\theta}{\tau}\right) h(\theta - \theta_0) d\theta}, \end{aligned}$$

where $h(\cdot)$ is the prior distribution of regime strength (centered at zero), and θ_0 is corresponding expected value.

The elite's joint posterior belief about θ and the behavior of the Mass Public is more involved:

$$P(\lambda < \theta \leq 1 \quad \& \quad x \leq x^*|y) = \int_{\lambda}^{1} P(\theta \quad \& \quad x \leq x^*|y) d\theta$$
$$= \frac{\int_{\lambda}^{1} P(y \quad \& \quad \varepsilon < \frac{x^*-\theta}{\sigma}|\theta) h(\theta - \theta_0) d\theta}{\int_{-\infty}^{\infty} P(y \quad \& \quad \varepsilon < \frac{x^*-\theta}{\sigma}|\theta) h(\theta - \theta_0) d\theta} = \frac{\int_{\lambda}^{1} F\left(\frac{x^*-\theta}{\sigma}\right) g\left(\frac{y-\theta}{\tau}\right) h(\theta - \theta_0) d\theta}{\int_{-\infty}^{\infty} F\left(\frac{x^*-\theta}{\sigma}\right) g\left(\frac{y-\theta}{\tau}\right) h(\theta - \theta_0) d\theta}.$$

Given these beliefs, the indifference condition for the opposition elite that determines the threshold y^* is given by:

$$\begin{aligned} &(1+\alpha)\left(\frac{\int_{-\infty}^{\lambda} g\left(\frac{y-\theta}{\tau}\right) h(\theta-\theta_0) d\theta}{\int_{\infty}^{\infty} g\left(\frac{y-\theta}{\tau}\right) h(\theta-\theta_0) d\theta} + \frac{\int_{\lambda}^{1} F\left(\frac{x^*-\theta}{\sigma}\right) g\left(\frac{y-\theta}{\tau}\right) d\theta}{\int_{-\infty}^{\infty} F\left(\frac{x^*-\theta}{\sigma}\right) g\left(\frac{y-\theta}{\tau}\right) h(\theta-\theta_0) d\theta}\right) \\ = &(1-\alpha)\left(\frac{\int_{1-\lambda}^{\infty} g\left(\frac{y-\theta}{\tau}\right) h(\theta-\theta_0) d\theta}{\int_{\infty}^{\infty} g\left(\frac{y-\theta}{\tau}\right) h(\theta-\theta_0) d\theta} + \frac{\int_{0}^{1-\lambda} F\left(\frac{\theta-x^*}{\sigma}\right) g\left(\frac{y-\theta}{\tau}\right) h(\theta-\theta_0) d\theta}{\int_{-\infty}^{\infty} F\left(\frac{\theta-x^*}{\sigma}\right) g\left(\frac{y-\theta}{\tau}\right) h(\theta-\theta_0) d\theta}\right). \end{aligned} \tag{35}$$

Similarly, the expected payoff to the Mass Public from mobilizing is

$$(1-\beta)(P(\theta \leq 1-\lambda|x) + P(1-\lambda < \theta \leq 1 \quad \& \quad y \leq y^*|x)),$$

and the expected payoff from refraining is

$$(1+\beta)(P(\theta > \lambda|x) + P(0 < \theta < \lambda \quad \& \quad y > y^*|x)).$$

The posterior belief of the Mass Public is analogous to that of the opposition elite:

$$P(\theta \leq 1-\lambda | x) = \frac{\int_{-\infty}^{1-\lambda} f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta},$$

and

$$P(1-\lambda < \theta \leq 1 \quad \& \quad y \leq y^* | x) = \frac{\int_{1-\lambda}^{1} G\left(\frac{y^*-\theta}{\tau}\right) f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{y^*-\theta}{\tau}\right) f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta}.$$

The indifference condition that determines the threshold x^* is as follows:

$$\begin{aligned} &(1-\beta)\left(\frac{\int_{-\infty}^{1-\lambda} f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta} + \frac{\int_{1-\lambda}^{1} G\left(\frac{y^*-\theta}{\tau}\right) f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{y^*-\theta}{\tau}\right) f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta}\right) \\ &\quad = (1+\beta)\left(\frac{\int_{\lambda}^{\infty} f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x-\theta}{\sigma}\right) h(\theta-\theta_0) d\theta} + \frac{\int_{0}^{\lambda} G\left(\frac{\theta-y^*}{\tau}\right) f\left(\frac{x-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{\theta-y^*}{\tau}\right) f\left(\frac{x-\theta}{\sigma}\right) d\theta}\right). \end{aligned} \tag{36}$$

Even though the model and the case analyzed here are fundamentally different, the effect of rents from power α on the endogenous variables seems to be the same (at least numerically) as in the model. As α increases, the threshold ys used by the opposition elite to decide whether to mobilize goes up—the elite is increasingly likely to mobilize ex-ante. The Mass Public sees in α a cue to infer the elite's behavior. A higher α increases the Mass Public assessment about the likelihood of regime change. Coordination therefore encourages the Mass Public to mobilize against regime. Figure 2 shows the numerical ex-ante probability of regime change for the case in which the Mass Public is a single player and compares this probability to the case in the model. Note that as α increases, the ex-ante probability of regime change increases in both models. Moreover, such probabilities have very similar values.

The patterns in Figure 2 do not seem to be coincidental (and suggest a unique equilibrium given by the equations above). Figure 3 illustrates the effect of various levels of information precision for the Mass Public (σ) and the size of opposition elite (λ). We numerically computed the ex-ante probability of revolution as a function of α when $\sigma = .5, 2$ and when $\lambda = 0.5, 0.75$ as well. Both graphs show that the ex-ante probability is increasing in α for

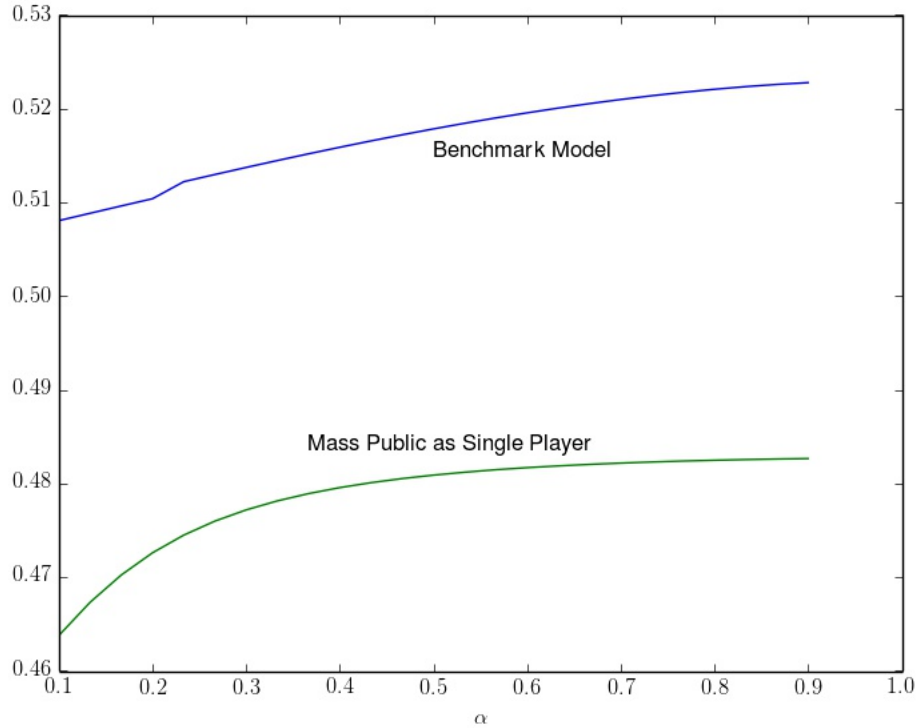

Figure 2: Ex-ante Probability of Revolution - Many Citizens vs Mass Public as Single Player

$\sigma, \tau = 1,\ \beta, \lambda = 0.1,\ \theta_0 = 0.5$

all the values of σ and λ considered.

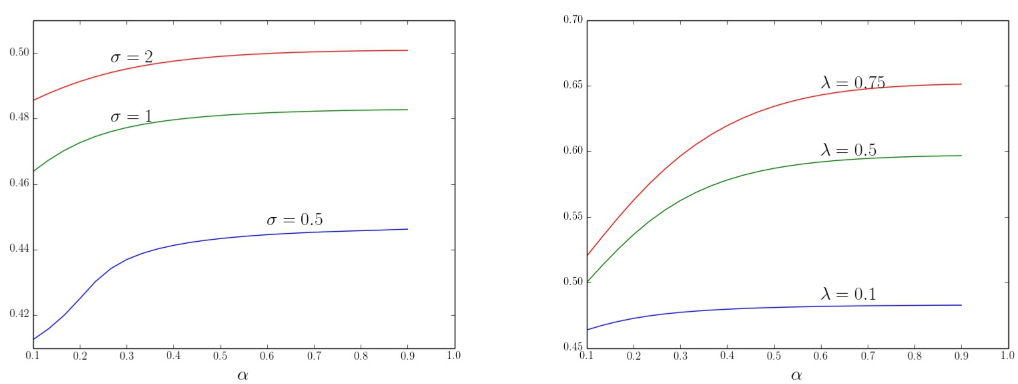

Figure 3: Ex-ante Probability of Revolution

$\tau = 1,\ \beta = 0.1,\ \theta_0 = 0.5$

5 Symbolic Leaders

In the sequential version of the model, we focus on the case in which the elite move first. The sequential case assumes that every player would ignore any player other than the elite moving first. We focus on the equilibria driven by this assumption because it seems a natural scenario if we think of the opposition elite as a prominent political player. However, it is possible to think about examples in which individuals who are not explicitly backed by sizable groups may take the initiative moving against the regime (e..g, the lone protester in Tiananmen Square).

We focus on the class of equilibria in which every player reacts to the information provided by the emergence of a symbolic leader. In this situation the elite does not gain anything by moving first, because moving first does not sway the mass public. As a result, the elite waits and learns from the symbolic leader's decision. In this class of equilibria, any citizen from the mass public is "designated" to move first and all the other players move afterward (in the "second period"). This implicitly assumes that taking the initiative is not conditional on the realization of the signal. In other words, we can think of the individual who takes the initiative to be selected before the game is played.

In this section we state the necessary conditions describing such class of equilibria. These conditions, however, do not allow us to derive analytic solutions for the equilibrium thresholds, so we rely on numerical calculations to compare the results obtained here with those in the model.

The symbolic leader receives the signal

$$z = \theta + \sigma\varepsilon.$$

Conditional on the realization of this signal, she will choose to mobilize (M) or refrain from mobilizing (R). The rest of the players play a simultaneous-move game similar to the one in the model except that now they condition their decision on the behavior of the symbolic leader in addition to their private signal. Accordingly, the play in the second period is characterized by eight thresholds: $\{\{y_M^*, x_M^*, \bar{\theta}_M, \underline{\theta}_M\}, \{y_R^*, x_R^*, \bar{\theta}_R, \underline{\theta}_R\}\}$, where the subscripts denote whether the symbolic leader mobilizes or refrains. Anticipating the behavior of the followers, the symbolic leader bases her decision on these eight thresholds. The condition that determines the threshold on the leader's private signal, z^*, is given by

$$(1-\beta)p_M = (1+\beta)(1-p_R), \tag{37}$$

where p_M is the posterior belief of the symbolic leader that regime change will occur given that she chooses M, and p_R is the probability of the same event given that she chooses R. Condition (37) parallels the indifference condition of the large player in the sequential version of the model. In both cases, the leaders face a trade-off between their biases favoring a particular option and the uncertainty about the outcome of the game. There are, however, two important differences between this case and the case in the model. Firstly, the preferences of the symbolic leader are not different from the preferences of all the other citizens. The symbolic leader exerts power over the elite through its effect on the mass public's decision. Secondly, the symbolic leader faces strategic uncertainty regarding the behavior of the elite in addition to the uncertainty about the strength of the regime θ. In the model, in contrast, the elite (acting as a leader) has different preferences and infer the behavior of the mass public perfectly by the law of large numbers. It is the strength of the regime its sole source of uncertainty.

After the symbolic leader chooses M (R), coordination on M can occur if a) the opposition elite choose M and θ is less than $\bar{\theta}_M$ ($\bar{\theta}_R$) or b) the opposition elite chooses R and θ is less than $\underline{\theta}_M$ ($\underline{\theta}_R$). The posterior beliefs of the symbolic leader in each case are given by

$$p_M = P(y \leq y_M^* \quad \text{and} \quad \theta \leq \bar{\theta}_M | z) + P(y \geq y_M^* \quad \text{and} \quad \theta \leq \underline{\theta}_M | z)$$
$$p_R = P(y \leq y_R^* \quad \text{and} \quad \theta \leq \bar{\theta}_R | z) + P(y \geq y_R^* \quad \text{and} \quad \theta \leq \underline{\theta}_R | z).$$

We calculate first p_M. The first term is given by

$$P(y \leq y_M^* \quad \text{and} \quad \theta \leq \bar{\theta}_M|z) = \int_{-\infty}^{\bar{\theta}_M} P(y \leq y_M^* \& \theta|z) d\theta$$

$$= \frac{\int_{-\infty}^{\bar{\theta}_M} P(y \leq y_M^* \& z|\theta) d\theta}{\int_{-\infty}^{\infty} P(y \leq y_M^* \& z|\theta) d\theta} = \frac{\int_{-\infty}^{\bar{\theta}_M} G\left(\frac{y_M^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{y_M^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}.$$

The second term is given by

$$P(y \geq y_M^* \quad \text{and} \quad \theta \leq \underline{\theta}_M|z) = \frac{\int_{-\infty}^{\underline{\theta}_M} G\left(\frac{\theta-y_M^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{\theta-y_M^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}.$$

Similarly, the two terms comprising p_R are given by

$$P(y \leq y_R^* \quad \text{and} \quad \theta \leq \bar{\theta}_R|z) = \frac{\int_{-\infty}^{\bar{\theta}_R} G\left(\frac{y_R^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{y_R^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta},$$

$$P(y \geq y_R^* \quad \text{and} \quad \theta \leq \underline{\theta}_R|z) = \frac{\int_{-\infty}^{\underline{\theta}_R} G\left(\frac{\theta-y_R^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{\theta-y_R^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}.$$

In sum,

$$p_M = \frac{\int_{-\infty}^{\bar{\theta}_M} G\left(\frac{y_M^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{y_M^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta} + \frac{\int_{-\infty}^{\underline{\theta}_M} G\left(\frac{\theta-y_M^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{\theta-y_M^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}$$

$$p_R = \frac{\int_{-\infty}^{\bar{\theta}_R} G\left(\frac{y_R^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{y_R^*-\theta}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta} + \frac{\int_{-\infty}^{\underline{\theta}_R} G\left(\frac{\theta-y_R^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} G\left(\frac{\theta-y_R^*}{\tau}\right) f\left(\frac{z-\theta}{\sigma}\right) d\theta}. \tag{38}$$

Given the action of the symbolic leader, the thresholds for θ are determined as in the model (these are the "critical mass" conditions):

$$\begin{aligned}(1-\lambda)F\left(\frac{x_M^*-\underline{\theta}_M}{\sigma}\right) &= \underline{\theta}_M\\ \lambda+(1-\lambda)F\left(\frac{x_M^*-\bar{\theta}_M}{\sigma}\right) &= \bar{\theta}_M\\ (1-\lambda)F\left(\frac{x_R^*-\underline{\theta}_R}{\sigma}\right) &= \underline{\theta}_R\\ \lambda+(1-\lambda)F\left(\frac{x_R^*-\bar{\theta}_R}{\sigma}\right) &= \bar{\theta}_R. \qquad (39)\end{aligned}$$

The large player's posterior probability about θ incorporates information from both her private signal and the publicly observable action of the symbolic leader. It is given by

$$\begin{aligned}P(\theta|y \quad \text{and} \quad z\le z^*) &= \frac{g\left(\frac{y-\theta}{\tau}\right)F\left(\frac{z^*-\theta}{\sigma}\right)}{\int_{-\infty}^{\infty} g\left(\frac{y-\theta}{\tau}\right)F\left(\frac{z^*-\theta}{\sigma}\right)d\eta},\\ P(\theta|y \quad \text{and} \quad z> z^*) &= \frac{g\left(\frac{y-\theta}{\tau}\right)F\left(\frac{\theta-z^*}{\sigma}\right)}{\int_{-\infty}^{\infty} g\left(\frac{y-\theta}{\tau}\right)F\left(\frac{\theta-z^*}{\sigma}\right)d\eta}. \qquad (40)\end{aligned}$$

The thresholds y_M^* and y_R^* ("cut-off" conditions) for the large player are given by

$$\begin{aligned}(1+\alpha)P(\theta\le\bar{\theta}_M|y_M^* \quad \text{and} \quad z\le z^*) &= (1-\alpha)P(\theta\ge\underline{\theta}_M|y_M^* \quad \text{and} \quad z\le z^*),\\ (1+\alpha)P(\theta\le\bar{\theta}_R|y_R^* \quad \text{and} \quad z> z^*) &= (1-\alpha)P(\theta\ge\underline{\theta}_R|y_R^* \quad \text{and} \quad z> z^*). \qquad (41)\end{aligned}$$

Finally, each citizen follower i forms his posteriors as follows:

$$\begin{aligned}P(\theta|x_i \quad \text{and} \quad z\le z^*) &= \frac{f\left(\frac{x_i-\theta}{\tau}\right)F\left(\frac{z^*-\theta}{\sigma}\right)}{\int_{-\infty}^{\infty} f\left(\frac{x_i-\theta}{\tau}\right)F\left(\frac{z^*-\theta}{\sigma}\right)d\theta},\\ P(\theta|x_i \quad \text{and} \quad z> z^*) &= \frac{f\left(\frac{x_i-\theta}{\tau}\right)F\left(\frac{\theta-z^*}{\sigma}\right)}{\int_{-\infty}^{\infty} f\left(\frac{x_i-\theta}{\tau}\right)F\left(\frac{\theta-z^*}{\sigma}\right)d\theta}. \qquad (42)\end{aligned}$$

Given these posteriors, the "cut-off" conditions for the citizens are determined as in the simultaneous case:

$$\frac{\int_{-\infty}^{\underline{\theta}_M} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{z^*-\theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{z^*-\theta}{\sigma}\right) d\theta} + \frac{\int_{\underline{\theta}_M}^{\bar{\theta}_M} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{z^*-\theta}{\sigma}\right) G\left(\frac{y_M^*-\theta}{\tau}\right) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{z^*-\theta}{\sigma}\right) G\left(\frac{y_M^*-\theta}{\tau}\right) d\theta} = \frac{1+\beta}{2},$$

$$\frac{\int_{-\infty}^{\underline{\theta}_R} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{\theta-z^*}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{\theta-z^*}{\sigma}\right) d\theta} + \frac{\int_{\underline{\theta}_R}^{\bar{\theta}_R} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{\theta-z^*}{\sigma}\right) G\left(\frac{y_R^*-\theta}{\tau}\right) d\theta}{\int_{-\infty}^{\infty} f\left(\frac{x_i-\theta}{\tau}\right) F\left(\frac{\theta-z^*}{\sigma}\right) G\left(\frac{y_R^*-\theta}{\tau}\right) d\theta} = \frac{1+\beta}{2}. \quad (43)$$

The “critical mass” and “cut-off” conditions together determine the equilibria. Although we are not able to obtain analytical solutions for the thresholds, we believe it is instructive to compare the symbolic leader case we discuss here with the sequential-move game discussed in the model at least numerically.

When the opposition elite mobilize in the first period, each individual citizen knows that a non-zero mass λ of the population have already mobilized against the regime, making it vulnerable against further mass public mobilization. Moreover, mobilization by the elite conveys to citizens a the regime is weak. Each citizen, however, has to consider that the opposition elite is also motivated by rents from power (α). Increasing rents from power renders the elite’s mobilization less informative about the actual strength of the regime. Therefore, as rents from power increase mobilization by the mass public goes down.

Although the symbolic leader adds negligible mass to the mobilization, her decision is influential as her payoffs are the same as those ones of the citizens in any contingency. Whenever she mobilizes against the regime, she sends a very strong signal to the rest of the citizens that mass mobilization is likely to be successful. Citizens do not discount their beliefs about mass mobilization as they do when the opposition elite move first. This is an additional source of power for the symbolic leader that the opposition elite does not have.

Figure 4 shows the size of the interval in which the leader is pivotal for each α, for each of the two cases analyzed (symbolic leader and the model in the paper). A small interval means that for most of the values of θ the resulting regime does not depend on whether the leader mobilizes. In other words, Figure 4 compares the importance of a symbolic leader and the opposition elite in the model in the paper. As it is explained in the paper, the interval for the model in the paper decreases in α as rents from power confuse the greed of the opposition elite with correct information about the regime's strength. In the symbolic leader case here, increasing α allows everyone else to entertain higher expectations about elite mobilization. Thus, this makes the symbolic leader to mobilize as well. Whenever the symbolic leader does not mobilize she conveys the regime is strong enough to cope with a highly incentivized opposition elite and everyone else seeking to coordinate with the elite.

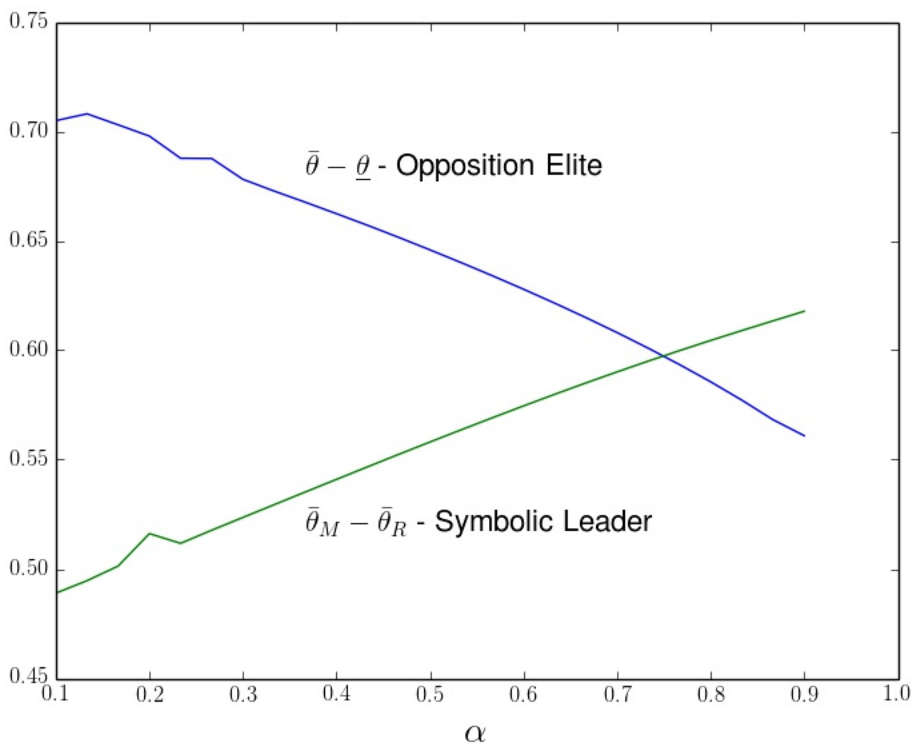

Figure 4: Pivotality of the Leader - Rents from Power

$\sigma, \tau = 1$, $\beta, \lambda = 0.1$

Figure 5 shows the size of the interval in which the leader is pivotal for each σ, for the symbolic leader case and the model in the paper. In the model, if the individuals' information is less precise than the opposition elite's, the latter is more effective in influencing the masses. In the symbolic leader case, on the other hand, the symbolic leader loses its influence over the mass public and the elite as the relative precision of her information falls. Therefore, as

Figure 5 illustrates, the region where the symbolic leader is pivotal shrinks.

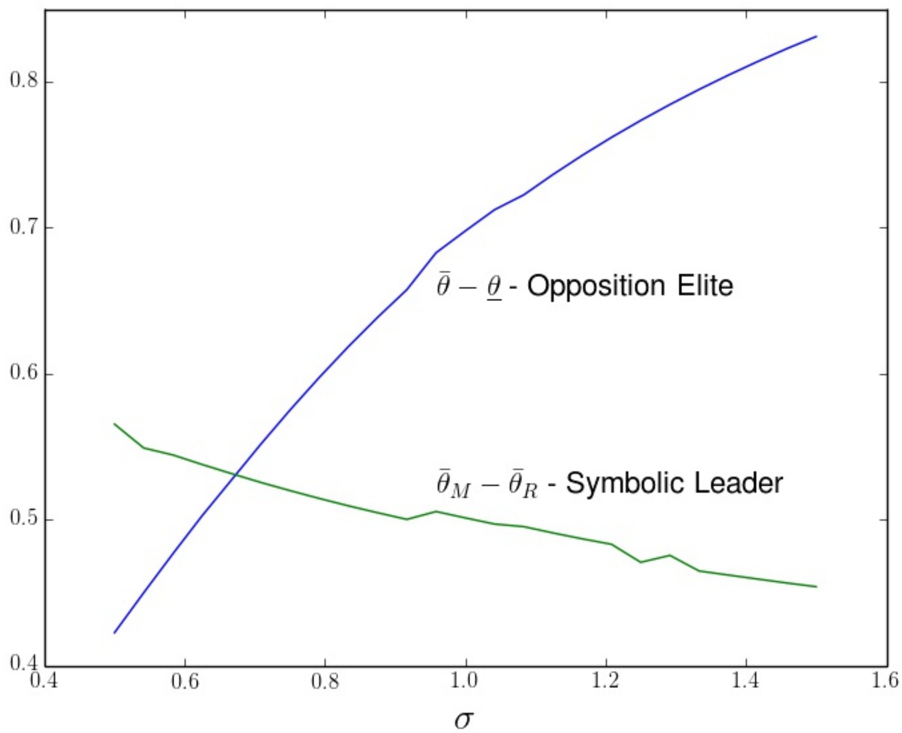

Figure 5: Pivotality of the Leader - Information Asymmetry

$\sigma = 1$, $\beta, \lambda = 0.1$, $\alpha = 0.2$

References

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