Consolidating Order and Prosperity: State Formation with Endogenous Military and Productive Capabilities

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Abstract

Jointly achieving order and economic growth must break away from a trap: the more growth is fostered, the stronger the incentives for challenging state control over resources. We study the trade offs facing the ruling elite of a proto-state on its path to growth and state consolidation. This study is relevant both for the problem of state formation in historical perspective and to the problem of state building in modern times. We distinguish between two forms of investment: in military capacity and productive capacity. We derive lessons for state-building by showing how investments in military capacity might have to precede investments in productive capabilities, and how a balance between the two may have to be maintained to ensure prosperity and peace. We show how economic shocks and arms innovations may trigger state consolidation and economic take-off or keep polities in a trap of political conflict and economic stagnation.

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1 Introduction

As attested in the anthropological literature, the rise of "civilization" –stratification, urbanization, monuments, writing, public architecture—was first observed nearly 5,000 years ago, and coincided with the emergence of the state organization (Trigger 2007). This emergence was probably the single most momentous transformation in social life since humans appeared nearly 200,000 years ago. States were preceded by populations settling in fixed locations where they adopted agriculture and developed food production (see, e.g. Frick and Meyers 1997: II: 15). These sedentary groups engaging in food production were able to generate and store surpluses. This allowed them to weather shocks, and increase their numbers. When they managed to reach the level of organization that is associated with states, they could finance the visible marks of civilization.

The advent of sedentism and food production, and of states afterwards, did not occur in a geographic and human vacuum. Food producers were surrounded by nomadic foragers and pastoralists who came to see the agricultural surplus as a tempting opportunity for expropriation. Intergroup violence had been prevalent since before civilization (Keeley 1996). The emergence of stored surpluses in the hands of groups tied to the land, who as a result could not flee, intensified the potential for conflict, prompting the settled populations to defend themselves. This conflict is seen as a central phenomenon by scholars that have analyzed the emergence of pristine states. As summarized by Mann (1986), "*The greater the surplus generated, the more desirable it was to preying outsiders. And the greater the fixity of investment, the greater the tendency to defend.*" The idea that the defense of the food-producing settlement was a central concern at the start of civilizations is attested by a striking architectural regularity in the archaeological record: virtually all ancient cities were surrounded by protective walls (Frick and Meyers 1997: II: 17).

The rise of civilization then involved a pair of core elements: generation and storage of surplus ("prosperity"), and the need to defend that surplus (to attain "security"). The dual importance of prosperity and security suggests a central paradox around state-backed civilizations. States are to produce security, but also require large surpluses, which in turn attract predation, undermining security. This is the puzzle of state formation: the steps to bringing about civilization intensified the forces conspiring against it.

A relevant fract to this puzzle is that prosperous societies could perhaps better finance the means to achieve security. Thus, the rise of a state requires moving from a situation without much security nor prosperity, to one with a sufficient degree of both. The central question in this paper is how, and under what conditions, such a transition could take place. We construct a formal model centered on the relation between productive and military capabilities, two key drivers of prosperity and security. We motivate the assumptions of our model by reference to the anthropological literature on early civilizations, and we then use the results of the model to account for the rise of pristine states, with focus on the first two civilizations in world history, namely Sumeria and Egypt.

In our model, we postulate the existence of an "incumbent" facing the potential attacks of a "challenger," which captures the historical case of budding urban centers (an agricultural settlement, or a port city home to traders) facing the predatory attacks of nomadic tribes or plundering warlords. In that setting, the incumbent has the opportunity to invest and grow future income, but may prefer to spend its resources in consumption and defense if future flows may be lost after a successful attack by the challenger. There are three fundamental parameters in the model, namely baseline income, and then 'military capacity' and 'productive investment capacity.' The two capacities are the rates at which current consumption can be transformed into defense and future income, respectively. All three parameters regulate fundamental tradeoffs. More baseline income helps finance more defense, but it also attracts stronger predation. The defense effort is necessary to prevail against the challenger and get to enjoy future income; if sufficient defense can be financed, the challenger can be (endogenously shown to be) deterred, guaranteeing the access to future income. If attacks will take place however, the effective rate of return to productive investments may not be high enough to justify investment. Moreover, by raising future income, productive investment may intensify predatory challenges, raising the need to defend.

We perform our analysis in two main parts. In the first part, we take the incumbent's military capacity as exogenous, and examine his decision over consumption, defense effort, and productive investment. In the second part we allow the incumbent to improve his military capacity–i.e., to increase the future rate at which he will be able to transform income into defense. Both parts are helpful in rationalizing the rise of ancient civilizational states.

The first main finding of our analysis with exogenous military capacity is that equilibrium induces a partition of the parameter space for military and productive investment capabilities, and that higher baseline income exacerbates predation. The partition yields four regions, showing that all four 'prosperity-security' combinations are theoretically possible. The relationship between security and prosperity has been a perennial concern in the social sciences. The standard Hobbesian view is that state-provided security is a precondition for prosperity. But that view misses the important fact that states themselves need to be explained, and may well require some prosperity to exist. When both military and investment capacities are low, polities remain in a stagnant and conflictive status as characterized by Keeley (1996), and typically associated with the pre-civilizational society that the Hobbesian state came to remedy. If investment returns are high relative to the polity's military capacity, growth is possible, but the polity must tolerate attacks. Although anti-Hobbesian, the possibility of prosperity without security is consistent with a widespread occurrence in the history of humanity: populations that prefer to grow their economies rather than building fully deterrent military protection despite threats of predation from neighboring plunderers, like the Chinese with Mongolians and the Saxons with Vikings in the 10th century. Lastly, when military capacity is high, and in particular it is high enough given the investment capacity of the economy, the polity can manage both to grow and deter predators. We take the last two analytic cases to be helpful in rationalizing the emergence of civilizations. These are situations where, enjoying high returns to productive investment, the economy grows above subsistence levels, and where the incumbent manages either a complete monopoly on violence by deterring the challenger, or where it mounts a defense strong enough so as to defeat the challenger with reasonably high probability (complete security is an elusive feature in the historical record).

In short, the type of parameter configurations compatible with states are those with high returns to productive investment and high military capacity. This conclusion illustrates how the model can account for the rise of states by mapping the parameter regions compatible with such a development to historical conditions. As noted by Johnson and Earle (2000), the rise of pristine states is strongly associated with the presence of alluvial agriculture. This activity, as we will argue later when studying in detail the cases of Egypt and Sumeria, implies two features: high baseline income and high returns to investments in productive infrastructure, mainly in terms of irrigation. The high returns to investment explain economic growth, but why did high baseline income not prevent consolidation by attracting predation? The answer is that pristine states also had a high military capacity. Egypt is perhaps the perfect example. Scholars emphasize that Egypt enjoyed a high natural defense due to the deserts surrounding the Nile valley (Bradford 2001)–a feature equivalent to an exogenously high military capacity. The combination of high returns to investment and high military capacity poised Egypt to attain prosperity and security, escaping the stagnation-conflict possibility.

The rise of civilization in Southern Mesopotamia poses a challenge to our model with exogenous military capacity, however. The reason is that, unlike Egypt, settlements in Sumeria that would go on to become the first city-states did not enjoy natural protection. Rather, as widely attested in the archaeological record, they faced recurrent attacks from various pastoralist groups that viewed these settlements as a tempting opportunity for predation. How could these first states ever emerge? The picture put forward by the anthropological literature is that the settled groups who developed pristine states exploited an agrarian "staple finance" that, being highly rewarding, would fund their states (Johnson and Earle 2000, p.305-306). These groups had enough of a material advantage that could be turned into a military one, by relying on walls, armor, weaponry, and numbers, all of which could be used to deter or defeat their enemies.

The question in terms of our model is whether the settlements in Southern Mesopotamia could use income to raise their initially low military capacity, and get to a parametric position similar to that which naturally occurred in Egypt. To answer this question we extend our basic model to allow for endogenous upgrades in military capacity. We show that a polity that is initially weak in terms of defense may be able to build its military capacity, and move into a parametric region where sufficient amounts of security and prosperity can be attained. The key condition is that the baseline level of income be high enough. This suggests an interrogation of the archaeological record to inquire whether Sumeria would be, of all places, one with a distinctly high baseline income. The fact is that Sumeria had, like the rest of Mesopotamia, access to an unusually favorable collection of plant and animal domesticates, which would yield a high baseline income compared to most other areas. An additional question is why the first states in history emerged in Southern rather than in Northern Mesopotamia. Like Egypt, Sumeria relied on alluvial agriculture, which offered substantially higher yields than agriculture in rain-fed areas such as those of Northern Mesopotamia. This gave Sumeria two things: high returns to investment, which our model highlights as a necessary condition, and higher baseline income with which to finance defense.

A second application of the model concerns the analysis of shocks to military and productive investment capacities; these shocks generate transitions from one area to another in the security/prosperity space. A rich picture of transitions where the effect of shocks depends on initial conditions emerges. In addition, such shocks can rationalize not only the rise of civilizations, but also their fall and the emergence of dark ages where security and prosperity are lost. We exemplify this type of application with a study of the end of the Bronze Age around 1200BC.

Despite our focus on the historical rise (and fall) of civilizations, applications to contemporaneous issues are also feasible. If exogenous shocks can shift a polity from one combination of security and prosperity to another, foreign policy interventions in failed states that recreate those shocks can do the same. Thus, we can apply our model to draw some lessons for state-building in modern societies featuring failed states. The defense community of the United States has reached a consensus around the idea that development initiatives can be an important element in counterinsurgency and state-building (see for instance the Counterinsurgency Manual from the Headquarters of the Army, 2006). An important question concerns the relative weight of development vs military build up initiatives in countries like Iraq or Afghanistan. Our model shows that enhanced military capacities are a necessary condition for achieving security and prosperity, while expanding investment capacities is not. Moreover, under certain conditions, an imbalanced mix may worsen outcomes.

The analysis of endogenous military capacity allows a different appreciation for the role of baseline income, and of income windfalls. While higher baseline income always exacerbates conflict in the model with exogenous military capacity, it may pave the way to security and prosperity when military capacity is endogenous. The new wealth is allocated to expanding military capacity in order to enable security and protect the economy. Once security is in place, a Hobbesian effect is observed: the new levels of defense prevent predatory challenges, and the enhanced return to productive investment ushers in growth.

The endogeneity of military capacity proves decisive to the role of income shocks at promoting, rather than hindering, state consolidation. The model can therefore offer an explanation for how commercial and banking developments that increased income in some European nations [in centuries XX] may have enabled rulers to consolidate their power, and why some nations, like Argentina in the 19th century, appear to have consolidated a state after substantial, trade-related, positive income shocks.

The plan for the paper is as follows. In the next section we quickly review related literature. Section 3 presents the basic model with exogenous military capacity. In section 4 we offer two historical applications of the model, namely the rise of the Egyptian state and the end of the bronze age. In section 5 we present the full version of the model with endogenous military capacity, and use it to account for the rise of states in Sumeria in section 6. Section 7 contains some applications to more modern issues, including policy toward failed states. We conclude in section 8.

Literature

Our paper contributes to the understanding of how a polity may move from a stateless, subsistence economy to one that enjoys order and prosperity. By order we mean the presence of a monopoly on violence that results in the lack of conflict, and by prosperity we mean having the ability to grow income, and thus escape subsistence. The productive asset controlled by our incumbent player could be land or a trading infrastructure, but also a state apparatus that helps expand (and tax) the economy's revenue. In this sense, the productive investments in our model can be interpreted as investments in state capacity as studied by Besley and Persson (2012). Their Chapter 4 studies the incentives of a controlling party to expand state capacity when there is conflict. An important difference in our model is that it includes a key tradeoff governing the dynamics of state capacity: the probability and intensity of conflict in our model is endogenous to the investments made by those controlling the state. In other words, our model helps understand the dynamics of state capacity when expanding those capacities make the state a more attractive booty for challengers.

Our paper is also related to the formal study of state consolidation. Powell (2012) offers a treatment where state consolidation happens exogenously, while Powell (2013) considers endogenous consolidation. The key difference is that in our model consolidation is studied in relation to investment and growth.

The rich array of consequences that economic shocks may have for peace and growth outcomes underscores the complexity of the interrelation between order and prosperity. This message of our paper joins an emerging emphasis on the conditionality of effects in conflict research. For example, Dal Bó and Dal Bó (2011) identified theoretically the diverging effects of economic shocks on conflict depending on the relative factor intensity of the productive and conflict activities. These effects have received empirical validation in the Colombian context (see Dube and Vargas 2008). Using a cross-country approach, Besley and Persson (2010) showed that price shocks may have different effects on conflict depending on whether they affect exports vs. imports.

The Basic Model

Our baseline model features the incentives to build an army to protect wealth from usurpers at the cost of detracting from the resources that are available for consumption or investment. Later on we introduce the decision to invest in military capacity.

Players

There is an "incumbent" who controls a productive asset that yields a nonstorable flow $v_t > 0$ every period. The incumbent may be seen as the merchant elite of a port city who control the port infrastructure and the gains from trade. Alternatively, the incumbent can be seen as the ruling caste of a city controlling an agricultural hinterland. Thus, the productive asset in these examples is alternatively the port and an area of productive land. There is also a "challenger" who receives an exogenous income flow from nature that we normalized to zero, and who is interested in wresting control of the productive asset away from the incumbent. This challenger may be seen as nomadic tribes that threaten with invading the city (as with the Mongolians in China) or relatively idle men led by provincial warlords (as with caudillos in 19th century Argentina who threatened the wealthy port city of Buenos Aires).

Actions, resources and technology

In each period the incumbent can spend its flow v_t in consumption, productive investment i_t or mobilizing resources to defend its asset. One dollar of productive investment i_t costs one dollar of consumption and it adds $\rho > 1$ dollars to the yield of the productive asset in the future. That is, the asset yield evolves according to the relation $v_{t+1} = v_t + \rho i_t$; we abstract from depreciation for simplicity.

The effectiveness of the incumbent's army is denoted a_t and such an army costs the challenger an amount $\frac{a_t}{\kappa}$ where $\kappa \geq 0$ is the value of the incumbent's military capacity. The higher the military capacity of the incumbent, the higher the "firepower" a_t attained by a given war effort $\frac{a_t}{\kappa}$ (or alternatively, given a war effectiveness a_t, the lower is the war effort $\frac{a_t}{\kappa}$ when the military capacity κ is higher). In this section κ is exogenous and we will derive implications for conflict and growth stemming from different values of κ. The expanded version of the model in section 2.2 will be devoted to endogeneizing κ. Thus, in period t the incumbent must observe a budget constraint

$$v_t - i_t - \frac{a_t}{\kappa} \geq 0. \tag{1}$$

The challenger observes the choices of a_t and i_t by the incumbent and chooses its own war effort b_t.[1] If victorious in the first period the challenger captures control of the productive

[1] Assume that the challenger's war expense is basically effort is equivalent to assuming that the challenger's income is sufficient to finance the optimal war effort b_t^*. Because the effects of interest are not driven by a budget constraint on the challenger being binding, we follow the most parsimonious approach of not making explicit a resource constraint on the challenger.

asset in the second period. Whenever the challenger attacks ($b_t > 0$), it prevails with probability $\frac{b_t}{a_t+b_t}$ and it gains nothing with the complementary probability; in other words, we adopt the typical Tullock contest success function. If the incumbent is defeated it obtains an outside payoff normalized to zero; the challenger becomes the new incumbent and in the following period faces a new challenger. If the challenger selects $b_t = 0$ we say the incumbent has successfully deterred the challenger, and this lack of challenge to the authority of the incumbent is the outcome we associate with state consolidation.

Timing

In each period the incumbent selects a_t and i_t. After observing (a_t, i_t) the challenger selects b_t. If $b_1 = 0$, the players retain their positions in period 2. If $b_1 > 0$, then there is a war at the end of period 1. The winner of the war becomes the incumbent in period 2, and faces a new challenger then.

Payoffs - problems for the challenger and incumbent

Both challenger and incumbent are risk neutral and care linearly about income and units of effort. The incumbent acts as a Stackelberg leader, choosing a_1 and i_1 to maximize the value of the game for an incumbent V_t:

$$V_t = v_t - \frac{a_t}{\kappa} - i_t + \frac{a_t}{a_t + b_t} V_{t+1}. \tag{2}$$

The challenger chooses b_t to maximize the expression

$$W_t = \frac{b_t}{a_t + b_t} V_{t+1} - \frac{b_t}{\kappa_c}, \tag{3}$$

where $V_{t+1} = v_t + \rho i_t$, and where κ_c is the military capability of the challenger. We could also parametrize the challenger's objective with a factor h and write the expected benefit as $\frac{b_t}{a_t+b_t} h V_{t+1}$, so as to capture different levels of "hunger" by the challenger.[2] Although capturing a different substantive aspect, the parameter h would be mathematically redundant since the challenger's problem could be rewritten as involving a military capability of $h\kappa_c$ instead. Therefore we will abstract from the parameter h. To simplify notation, we will develop the model normalizing $\kappa_c = 1$, and will comment later on how the solution changes with variations in κ_c. An additional simplification is we do not consider here the realistic

[2] This parameter could also track the differential ability of the challenger at "operating" the asset. One issue we do not take up here is the case where a challenger has a high valuation for the stream of production (as when looting animals and food) but a low valuation for the asset due to an inability to operate it. These are interesting variations that go into the finer issue of modes of challenge that may be costly to the incumbent but do not pose a replacement threat. This is left for future research.

possibility that conflict destroys part of the asset. Our results do not change qualitatively by assuming that conflict is destructive, and we offer an extension in the appendix that covers such case.

We will solve for a Subgame Perfect Nash Equilibrium by backward induction.

1.1 Second period

In the second period, there is nothing the challenger would want to fight for, as there is no future in which to enjoy the productive asset if stolen. Thus, $b_2 = 0$. The incumbent then chooses i_2 and a_2 to maximize the value of consumption in the second period $V_2 = v_2 - i_2 - \frac{a_2}{\kappa}$. Since there is no use for an army and investment would only pay in a nonexistent third period, $i_2 = a_2 = 0$, yielding $V_2 = v_2$.

First period

The challenger observes the pair (a_1, i_1) and chooses b_1 to maximize W_1 as given by expression (3). Since the first order condition is $\frac{a_1}{(a_1+b_1)^2} v_2 = 1$, and $v_2 = v_1 + \rho i_t$, the best response function of the challenger is immediately seen to be,

$$b_1(a_1, V_2) = \begin{cases} \sqrt{a_1 (v_1 + \rho i_1)} - a_1 & if\, a_1 < V_2 \\ 0 & otherwise \end{cases}. \tag{4}$$

This expression exhibits a key trade-off of the model: productive investments i_1 raise the value of the productive asset. Thus, conditional on maintaining control of the asset, investment is a good idea for the incubment since $\rho > 1$; however, the future control of the asset is not a forgone conclusion. Investment raises the incentives of the challenger to arm itself since it makes it more attractive to become the incumbent. Therefore, while productive investments increase the value of future incumbency, they may lower the chance that the current incumbent gets to reap that value. The lack of state consolidation may be an obstacle for productive investment and growth. The fundamental problem is to understand whether there are any parameter values v_1, κ, and ρ that map into a path of state consolidation and growth. To answer this question we must study the problem of the incumbent.

The incumbent maximizes V_1 as given by (2) subject to the budget constraint (1) and anticipating the challenger's best response in (4). The latter indicates that if $a_1 \geq v_1 + \rho i_1$ the challenger will choose not to fight, and therefore the incumbent would never choose a_1 beyond the point $v_1 + \rho i_1$, which attains deterrence. This can be incorporated into the incumbent's problem as an additional deterrence constraint. The incumbent's problem in period one can then be written as,

$$\max_{a_1,i_1} v_1 - \frac{a_1}{\kappa} - i_1 + \frac{a_1}{a_1+b_1}(v_1+\rho i_1) \tag{5}$$

subject to

$$\begin{aligned} v_1 - \frac{a_1}{\kappa} - i_1 &\geq 0 \ (BC) &(6)\\ v_1 - a_1 + \rho i_1 &\geq 0 \ (DC) &(7)\\ a_1 &\geq 0\\ i_1 &\geq 0. \end{aligned}$$

Let us call λ_{BC}, λ_{DC}, λ_a and λ_i the lagrange multipliers for each restriction. The Lagrangian, which expresses the expected utility of the incumbent, is:

$$\begin{aligned} \mathcal{L} &= v_1 - \frac{a_1}{\kappa} - i_1 + \frac{a_1}{a_1+b_1}(v_1+\rho i_1)\\ &\quad +\lambda_{BC}(v_1 - \frac{a_1}{\kappa} - i_1) + \lambda_{DC}(v_1 - a_1 + \rho i_1) + \lambda_a a_1 + \lambda_i i_1. \end{aligned} \tag{8}$$

We will characterize the solution $(a_1, i_1, \lambda_{BC}, \lambda_{DC}, \lambda_a, \lambda_i)$ to this problem for each parameter combination (ρ, κ, v_1).

The first order and complementary slackness conditions that characterize the optimum are given by,

$$\frac{\partial \mathcal{L}}{\partial a_1} = \frac{1}{2}\sqrt{\frac{v_1+\rho i_1}{a_1}} - \frac{1}{\kappa} - \frac{\lambda_{BC}}{\kappa} - \lambda_{DC} + \lambda_a = 0; a_1 \geq 0, \lambda_a \geq 0, \lambda_a a_1 = 0 \text{ c.s.} \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial i_1} = \frac{\rho}{2}\sqrt{\frac{a_1}{v_1+\rho i_1}} - 1 - \lambda_{BC} + \lambda_{DC}\rho + \lambda_i = 0; i_1 \geq 0, \lambda_i \geq 0, \lambda_i i_1 = 0 \text{ c.s.} \tag{10}$$

$$\lambda_{BC}(v_1 - \frac{a_1}{\kappa} - i_1) = 0 \text{ c.s.}, \ \ \lambda_{DC}\,(v_1 - a_1 + \rho i_1) = 0 \text{ c.s.} \tag{11}$$

Solving the program (8) requires checking which combinations of values for the endogenous variables $(a_1, i_1, \lambda_{BC}, \lambda_{DC}, \lambda_a, \lambda_i)$ constitute the optimum for different regions of the parameter space $(\kappa, \rho, v_1) \in \mathbb{R}^3_+$. Note from (9) that the marginal benefit of a_1 goes to infinity as a_1 goes to zero (a typical feature of contests), so the optimum must feature $a_1 > 0$

and $\lambda_a = 0$. Beyond this, the method for solving the problem is tedious: it requires checking which combinations of values for the endogenous variables are consistent with the constraints for each parametric region and then identifying the ones that yield the highest value for the program. The details of the solution are contained in the appendix. A summary of the solution is offered in the following,

Proposition 1 *Optimal behavior by the incumbent yields a division of the parameter space* $(\kappa_1, \rho, v_1) \in \mathbb{R}^3_+$ *into four distinct regions:*

Region 1 (R1): $\left\{(\kappa, \rho, v_1) \in \mathbb{R}^3_+ | \kappa_1 \geq \rho,\ \rho \geq \kappa/(\kappa-1)\right\}$ *Consolidation and growth*

In R1 the solution is: $\left\{a_1 = v_1 \frac{\kappa(1+\rho)}{\kappa+\rho}, i_1 = v_1 \frac{1}{2}\left(1 - \frac{1}{\rho}\right), \mathcal{L} = v_1 \frac{1}{2}\left(1 + \frac{1}{\rho}\right)\sqrt{\rho\kappa}\right\}$

Region 2 (R2): $\left\{(\kappa, \rho, v_1) \in \mathbb{R}^3_+ | \rho > \kappa,\ \rho \geq 4/\kappa\, and\, \rho \geq 1\right\}$ *Unstable growth*

In R2 the solution is: $\left\{a_1 = v_1 \frac{\kappa}{2}\left(1 + \frac{1}{\rho}\right), i_1 = v_1 \frac{\kappa-1}{\kappa+\rho}, \mathcal{L} = v_1 \frac{1}{2}\left(1 + \frac{1}{\rho}\right)\sqrt{\rho\kappa}\right\}$

Region 3 (R3): $\left\{(\kappa, \rho, v_1) \in \mathbb{R}^3_+ | 2 \geq \kappa\, and\, \rho < 4/\kappa\right\}$ *Conflict and stagnation*

In R3 the solution is: $\left\{a_1 = v_1 \left(\frac{\kappa}{2}\right)^2, i_1 = 0, \mathcal{L} = v_1\left(1 + \frac{\kappa}{4}\right)\right\}$

Region 4 (R4): $\left\{(\kappa_1, \rho, v_1) \in \mathbb{R}^3_+ | \kappa_1 \geq 2,\ \rho < \kappa_1/(\kappa_1 - 1)\right\}$ *Stagnant consolidation*

In R4 the solution is: $\left\{a_1 = v_1, i_1 = 0, \mathcal{L} = v_1\left(2 - \frac{1}{\kappa}\right)\right\}$.

Proof: See appendix.

The following figure contains a graphical representation of the solution.

The payoffs that the incumbent obtains in each region are easily computed by noting that his expected utility at the beginning of period 1 is $v_0 - \frac{a_0}{\kappa_0} - i_0 + \frac{a_0}{a_0+b_0}V_1$ and that we .

Discussion

1.1.1 Properties of equilibrium

Internal conflict or external?

Is incumbent a ruler or the population? Is there internal oppression or benevolence?

A brief preamble on parameter interpretation [MODIFY THIS SUBSECTION TO REFLECT SUMERIA/EGYPT TYPE OF PARAMETERS INTERPRETATION]

The parameter v_1 tracks properties of the environment (e.g., weather, quality of the soil, topography) that affect the quantity of goods that the economy can produce. If the polity trades, v_1 will also be affected by the price fetched by the goods sold. A convenient feature of this model is that v_1 is a scale parameter and that the optimal decisions by the incumbent on defense a_t and productive investment i_t are invariant in v_1. (In later stages the parameter v_1 can be studied as a determinant of decisions to invest in improving productivity parameters,

κ or ρ). This feature greatly simplifies the characterization of emerging "regimes" as we can restrict attention to the bidimensional space (κ, ρ). A brief preamble on how to interpret these parameters is worthwhile at this point.

The two parameters $(\kappa\,, \rho)$ track respectively the productivity of expenditures in defense and productive investment. Thus, ρ captures anything that affects the returns to productive investments in the asset controlled by the incumbent. For example, ρ could, like v_1, respond to climatic conditions and other features of the environment, or to the price of goods sold.[3] As for κ, it captures anything that yields the incumbent an advantage at producing military firepower at a given expense, such as better military technology or expertise. Note however that changes in some types of infrastructure (e.g., introducing railroads) may affect both ρ and κ: a railroad may increase the returns to investing in a port, and it may also make the incumbent's army more effective.

Characteristics of the equilibrium: the effect of economic shocks

Given that for $\rho < 1$ investment is never worthwhile, failure to obtain it in equilibrium is obvious and uninteresting. We will focus on the area where $\rho > 1$ where investment is a possibility. The main feature of the solution is that all four combinations of order and prosperity can be observed depending on the values of the parameters (κ, ρ). For low values of both military capacity and yield of investment, the polity will be stuck in a situation of economic stagnation and conflict ($R3$). In $R3$ the prospect of conflict lowers the rate of return to investment preventing growth from occurring. If military capacity κ is low, an increase in the yield of productive investment is not guaranteed to help much (the region $R3$ extends upwards). If large enough, increasing the yield to productive investment may move the polity to a region ($R2$) where productive investments, and economic growth, occur, but conflict remains: investment yield ρ may be high enough that despite the possibility of conflict the incumbent wants to invest, but the appeal of seizing the asset fuels the challenger's incentives to fight. Only an even larger military capacity κ_1 would move the polity from $R2$ to $R1$, where peace is gained and higher investments and faster growth will occur.[4] Thus, gaining order in addition to the incipient prosperity of $R2$ further enhances prosperity.

[3] If the value of what the polity produces follows a standard price $\times$ quantity formulation we can write $v_1 = p.q$, and $v_2 = v_1 + \rho.p.i = p.q + \rho.p.i$, where q and i are physical units. Then, changes in p can be captured in our model as changes in both v_1 and ρ. Changes in the baseline physical capacity of production q will be captured through changes in v_1 exclusively, and changes in the physical returns to investment as changes in ρ only.

[4] Productive investment is higher in $R1$ than in $R2$ whenever $v\frac{c-1}{c+\beta} > \frac{v}{2}\left(1 - \frac{1}{\beta}\right)$ or when $\beta c - \beta + c - \beta^2 > 0$, which is always the case for $c_1 > \beta$, a condition characterizing $R1$.

As said earlier, the (order, prosperity) regimes characterized in Proposition 1 are invariant in initial income v_1; that is, whether consumption, investment i_1, and arming by the challenger b_1 are positive or zero does not depend on v_1. But changes in income v_1 do affect the particular values of all endogenous variables whenever positive. In particular, we have the following,

Proposition 2 *Increases in initial income exacerbate conflict; that is, in regimes where a_1 and b_1 are positive, they increase with v_1.*

Proof: see appendix.

In the world where military capacity κ is exogenous, by inspection of the expressions for i_1 in Proposition 1 we see that an income windfall will increase investment. However, from the last proposition we learn that income windfalls can only worsen conflict by exacerbating predatory challenges and defensive efforts.

A salient aspect of the model is that shocks to prices or technology that map into increases in ρ may have very different consequences depending of the value of military capacity. The following remark is a corollary to Proposition 1.

Corollary 1 *If $\kappa \leq 2$ and the polity is coping with disorder and stagnation in $R3$, an increase in ρ will keep the polity in the same region or make it transition into disorder and growth in $R2$. But if the polity has larger military capacity $\kappa > 2$ and faces peace with stagnation in $R4$, an intermediate increase in ρ could attain both order and prosperity (in $R1$), while a larger increase in ρ could make the polity jump to a regime of growth with disorder.*

The last corollary says that an improvement in the rate of return to investment can have different effects depending on the level of military capacity. The fact that the "sweet spot" $R1$ is wedge-shaped yields a central insight for state-building,

Remark 1 *From a situation of stagnation and conflict, a large enough increase in military capacity κ is a necessary and sufficient condition for successful state-building featuring consolidation and growth*

Starting from $R3$, the only way to conquer peace is to augment the military capacity of the incumbent. For moderate increases, peace may be conquered (moving into $R4$), but the cost of the arms that attain deterrence is still high enough that the incumbent cannot channel resources toward investment. Thus, peace is attained but the polity remains stuck in a no-growth regime. In $R3$ the fact that the incumbent may lose power acts as a tax on the returns to investment. In $R4$ the effect that prevents investment is more subtle: the incumbent realizes that investments will expand the voracity of the challenger, and this will trigger too high costs of maintaining the peace. Only large enough increases in military capacity, or a balanced increase in military capacity and investment yield, allow the incumbent to arm to levels that attain deterrence while leaving resources available for promoting investment and growth (a move from $R3$ to $R4$ and then to $R1$). Summarizing,

Remark 2 *Increases in the yield to productive investment ρ are not necessary nor sufficient condition for successful state building.*

Note that shocks may increase or decrease parameters like ρ and κ_1. A polity that enjoys order and prosperity in $R1$ with $\rho < 2$ could, through a reduction in κ, be plunged into stagnation and disorder in $R3$. A reduction in κ could be thought of as a negative shock to the incumbent's military technology.

1.2 Further results

1.2.1 Destructive conflict

As said earlier, this simplified model does not incorporate the destruction of resources brought on by conflict and it represents the limit solution to a more general model where only a fraction σ of the asset survives the war. The solution to the expanded model is similar and one can show that for σ low enough, an increase in ρ that moves the polity from a point of consolidation ($R4$ or $R1$) to one with conflict (in $R2$) may leave the polity worse off (see Appendix).

1.2.2 Changes in parameters of the challenger

In developing the basic model we normalized to 1 the parameter κ_c that captures the military capacity of the challenger. Recall that shifts in this parameter also capture shifts in a parameter that affects the challenger's valuation of the productive asset under control of the incumbent. In this subsection we investigate how shifts in κ_c affect the partition of the parameter space derived in Proposition 1.

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2 Historical applications

2.1 Egypt and the birth of a state

Among the first civilizations, Egypt is the prototype of a "pristine territorial state," the undisputed pioneer in attaining both order and prosperity throughout a substantial territorial expanse. Although the "Neolithic Revolution" occurred in Egypt much later than in Mesopotamia, the ensuing process of social and political development in Egypt was extraordinarily fast.[5] In less than a thousand years, the outcome would be a state that not only presided over a wealthy economy but was also able to protect its territory and the surplus generated within it for long stretches of time. As a specialist on Egyptian state formation put it, "*the Egyptian state lasted longer and was more stable than most Empires established elsewhere.*" (Allen 1997: 135).

Although the specific conditions underlying Egypt's dual economic and political evolution are disputed, a strong consensus exists around the general idea that Egypt's geography, the physical attributes of its territory, played a key role in the emergence of civilization. Our model can be used to identify three features of the Egyptian geography—the delta river, the potential of irrigation, and the surrounding desert—that collectively set it apart from other civilizations, and to assess their role in explaining the consolidation of a successful state.

(1) The Nile River as a fundamental driver of the Egyptian economy. The Nile had at least two key properties: a yearly flood that fertilized the soil with rich stilt from Ethiopia, and a two-way navigability that facilitated exchange along the entire valley.[6] "*[T]he Nile was perfectly ordered—its current carried boats downstream, the wind blew them back upstream—and the Nile's regular flooding renewed the fields and made farming so easy that in the Delta men had 'only to throw out seeds to reap a crop.'*"' (Bradford 2001: 9)) Both properties,

[5] During the Neolithic "revolution," gathering and hunting were gradually replaced by the domestication of plants and animals for food production. The process began in the ancient Near East (Southern Levant) about 10,000 years ago. The Neolithic in Egypt developed much later, around 5500 BC. According to Bard, "The beginning of the First Dynasty was only about 1000 years after the earliest farming villages appeared on the Nile, so the Predynastic period, during the 4th millennium B.C., was one of fairly rapid social and political evolution" (1994: 267).

[6] Most of the flow originated from monsoons in the Ethiopian highlands, and a smaller part came from the upper watershed of the White Nile around Lake Victoria. With impressive precision, the river began to rise in the South in early July and the flood got to the northern end of the valley by mid September. The Tigris and Euphrates were not only much less predictable in timing, but also much more irregular and less benign in volume.

natural fertility and easy exchange, map into a high v in our model, whereby available income is high even before investments are made.

(2) The productivity of artificial irrigation. Egyptians could vastly increase their economic output by investing in water management, which in the Nile valley took the form of basin irrigation. Egypt developed a network of earthen banks in the agricultural fields that would form a grid of basins to trap the floodwater and hold it for much longer than it would naturally stay. The basins allowed the earth to become fully fertilized before planting, and a system of canals redirected the remaining floodwater to basins in need.[7] Economic sociologists agree that in Egypt irrigation agriculture "*could generate crop-to-seed yield of between 12:1 and 24:1 ... but only at the cost of high capital investments*" (Morris and Manning 2005: 141). In Michael Mann's genealogy of social power, artificial irrigation involved one of the earliest forms of economic investment, which in Egypt was even more productive than in Mesopotamia. Both in Egypt and Mesopotamia, irrigation agriculture could "*generate a surplus far greater than that known to populations on rain-watered soil*" (1986: 80). In Egypt, "*the process was as in Mesopotamia, but squared,*" and "*as productivity grew, so too did civilization, stratification, and the state*" (1986: 108). In our model, a high value of the parameter rho reflects the existence of an environment in which investments yield large increases of economic surplus in the same way that the construction of irrigation systems resulted in major expansions of the food surplus in Egypt.

(3) Territorial isolation as natural protection. The Nile basin is surrounded by deserts, which made invasions much less likely than in other centers of civilization. According to Bradford, "*The sea to the north and the deserts west and east isolated the Egyptians from the rest of mankind, except for merchants, some infiltrators, and the occasional raid.*" (2001: 9). The desert provided two kinds of protection. On the one hand, the desert's inhospitality to human settlement discouraged the emergence of hostile neighbors. On the other, the desert was a formidable barrier against distant rivals. In terms of our model, Egypt's territorial isolation can be interpreted as a naturally high κ.

How do these conditions account for Egypt's twin achievements of order and prosperity in the context of our model? A high level of v has no effects in the model with exogenous military capacity (we will explore endogenous military capacity later). The implication is

[7] According to a long scholarly tradition, water management and state formation were closely linked in ancient societies. The thesis of "hydraulic empires," which claims that irrigation was a public good with enormous fixed costs, and that pristine states formed precisely in order to provide them, has been discredited by evidence showing that irrigation preceded (rather than resulted from) the creation of public administrations.

that the Nile's extraordinary natural fertility was not a decisive factor. Super-productive soils are not unique to Egypt. But the model does highlight that a combination of a high kappa and a high rho helped Egypt to attain prosperity and security. Protected by the deserts, Egypt had very few challengers against which its military capacity would be small. This would permit Egypt to fend off well against nomadic raiders interested in capturing Egypt's agricultural assets and surplus. Given the potential for irrigation infrastructure that would increase productivity, Egyptian rulers would have incentives to encourage investments that would increase future surplus.

The resulting picture is one where Egypt is located in a "favorable" section of region R2, if not directly in R1. The reason to place Egypt during the state formation period (end of the Naqada II period, around 3200BC) in a good part of R2 is that Egypt did face occasional attacks, and perhaps the total absence of challenges that characterizes R1 is better reserved to the heights of Egyptian power under the new kingdom, when the Egyptian state was even more dominant than during its formative phase. A "good" part of R2 is one near the frontier with R1, where kappa is so high that the arming that is done by the incumbent is small, and hence of little cost, and is still sufficient to guarantee victory with very high probability. Thus, a polity in such "good" part of R2 would grow and enjoy a relatively secure existence, because the probability of defeat is small, and the returns to investment are high. These conditions –high kappa, high rho—continued to prevail in Egypt for a long time, perhaps explaining the remarkable stability of the Egyptian policy alluded to earlier.

2.2 The end of the bronze age.

Compared with the rise of the great civilizations of the Bronze Age, the demise was a much more sudden affair: a wave of political and economic collapse swept across the Eastern Mediterranean around 1200BC, with drastic consequences in the Near East, Anatolia, Greece, and Egypt. An irreversible legacy of the collapse was the extinction of dozens of cities that were at the very frontier of political and technological development. Few events in history provide such a large scale and definite proof against unilinear visions of social progress.

For a period of almost 400 years, the Eastern Mediterranean had seen the rise of multiple states that improved their productive capacity and were capable of defending their wealth against "barbarian" populations. Progress on the productive dimension included advanced irrigation and plowing techniques for expanding agricultural surplus, storage facilities especially conditioned for the preservation of cereals, and permanent bureaucracies for economic redistribution. Military power was sustained by chariots and fortified walls. In combination, these achievements backed a sophisticated division of labor that allowed for the emergence of specialized ceramic and metal craft, and the perfection of writing, religion and the arts. This set of thriving states included the city-ports of the Levant, the kingdoms of Anatolia, the Egyptian empire, and the city-states of Mesopotamia and Cyprus.

"But then, the world as they had known it for more than three centuries collapsed and essentially vanished" as Eric Cline puts it (2014: 241). According to the assessment of Drews, "altogether the end of the Bronze Age was arguably the worst disaster in ancient history, even more calamitous than the collapse of the western Roman Empire" (1993: 3). The proximate cause of the end of the Bronze Age was, in most areas, invasion by armed forces coming from beyond civilization. The "Sea Peoples," as the Egyptians called them, were actually a diverse array of intruders with different geographic and ethnic origins (Sandars 1987), including the Deniens (either Greeks or the Dan tribe among Israelites), the Sherden (possibly Sardinians), the Shekelesh (Sicialians), the Lukka (from the Anatolian Aegean), and the Teresh (possible ancestors of the Etruscans).

There has been a long debate on the fundamental causes behind the fall of Bronze Age. Since the beginning of the debate in the mid-1960s, archaeologists have hypothesized that the collapse was set in motion by earthquakes (Schaeffer 1948), droughts and famines (Carpenter 1968), internal rebellions (Zuckerman 2007 and Carpenter 1968), or innovations in military technology (Drews 1993).

Our model can help think about the end of the Bronze Age in two ways, one particular and one general. The particular application, as we will see, is to show that most of the fundamental explanations given for the end of the Bronze Age can be analyzed, using our model, as shocks either the value to the challenger of the incumbent's asset (the parameter h), or the effectiveness of the incumbent's military investments (the parameter kappa). These interpretations, in addition, are consistent with the wealth of archaeological findings collected since the mid-1980s, which has tended to reinforce the notion that military struggle was involved in the process at least as a proximate cause for a high proportion of cases.

The more general point resulting from our use of the model is to relate changes in deep military and economic fundamentals to the arguments made by social theorists that the evolution of political complexity is not unilinear, but plagued by dead ends and reversals. According to our model, a few deep economic and military fundamentals shaped the equilibrium in the confrontation between the civilized centers and the "barbarian" periphery. A shock to any one of those fundamentals could shift societies from one combination of levels of order and prosperity to another combination. Importantly, those movements do not necessarily go in the direction of greater order and prosperity. The end of the Bronze Age involved state de-consolidation and a regression to lower income levels—a Dark Age–, , as in the region of conflict and stagnation, R3, in our model.

Debate among archeologists around the end of the Bronze Age has made true empirical and theoretical progress. Only two hypothesized causes are incompatible with the notion that the end of the Bronze Age involved a major military defeat of the civilized world: earthquakes and internal rebellions. Both explanations face challenges. The hypothesis of earthquakes has been discredited in the face of new archaeological evidence showing that most urban destruction was caused not by natural forces but by an enemy attack, which in the case of the key city of Ugarit left numerous arrow-heads throughout the ruins (Yon 1992: 117; Singer 1999: 730; Kanievski et al 2011). Attacks not only occurred but were endemic, even if un-coordinated: from opposite ends of the Bronze Age world, Hittite and Egyptian rulers left unequivocal testimonies about the menace and calamities of the incursions by the Sea Peoples, both in pictorial and written form.

The hypothesis of internal rebellions relies on the least plausible premise given the geographical scope and speed of the collapse: an extraordinary level of simultaneity among the rebels across the different sites of Anatolia, Greece, Northern Mesopotamia and the Levant. One potential driver of the simultaneity could be a common climatic shock.[8] But a more serious challenge to a pure internal rebellion story is posed by the fact that there is evidence of large migration movements across Late Bronze Age civilizations. This evidence is more compatible with invasions and exoduses than with simultaneous infighting.

The theoretical problem with the invasions is, of course, what caused them in the first place. Two hypotheses consistent with available evidence are:

(1) A severe change in climate, which caused draught and famines, and compelled the populations living beyond the gates of civilization to invade in search for food. Cities that were storehouses of grain fell victim to "a final resort to violence by a drought sicken people" (Carpenter 1968: 69).

(2) A revolution in the means of war, including the introduction of the javelin, which

[8] A recent paleobotany study based on samples of pollen throughout the Bronze and Iron Ages confirms the existence of a substantial climate change around the time of the collapse, which caused a reduction in precipitation and resulted in the shrinkage of the Mediterranean forest (Langgut, Finkelstein, and Litt 2013). In the interpretation of these authors, climate change and the ensuing famine may have caused internal rebellions rather than foreign invasions.

tipped the military balance in favor of nomadic intruders from economically less developed regions. Thus, according to Drews, "the Catastrophe was the result of a new style of warfare that appeared toward the end of the thirteen century BC, [which] opened up new and frightening possibilities for various uncivilized populations that until that time had been no cause of concern to the cities and kingdoms of the eastern Mediterranean" (1998: 33). What were the changes introduced by the "uncivilized pupulations"? Chrissantos summarizes them: "these tribes developed better and lighter body armor, [. . .] lighter and smaller round shields, [and] revolutionary longer, stronger swords [. . .] They also invented a new weapon, the javelin, which could be used as a missile to hurl at an enemy. They [managed to] overcome the civilizations' chariot advantage [...] Once these tribes mastered sea travel, no shore was too far for an attack. The failure of the chariot in the face of this new warfare marks the beginning of the Bronze Age world's collapse" (2008: 11).

At the theoretical level, of course, climate change and military innovation are not mutually incompatible causes and can be combined under the form of a "Perfect Storm" (Cline 2014: Chapter 5). Another recent theoretical development for combining ineliminable causes builds on the idea of "System Collapse." The point of departure is the fact that Late Bronze Age societies—kingdoms, villages, cities and empires—were all connected through frequent commercial, production, and diplomatic relations. The assumption is that such relations had reached such a level of intensity, specialization and complementarity that if the economy in one of them were to come to a halt, for whatever reason, the whole Eastern Mediterranean would collapse under "domino" and "multiplier" effects. This allows for the theoretical possibility that weather- and technology-induced invasions had devastated a critical number of nodes in the workings of the global Eastern Mediterranean, which eventually provoked its general collapse.

Our incumbent-challenger model is compatible with all surviving interpretations for the collapse of the Bronze Age, taken individually or in any of their combinations (general wave of invasions, invasions in critical sites, invasions prompted by climate-induced famines, invasions caused by changes in the art of war). More importantly, however, the definition of the incumbent's income generating asset v, and the productivity of its military investment kappa, helps to make a crucial distinction between two separate issues at play in the invasions that put an end to the civilized Eastern Mediterranean: the motivation versus the effectiveness of the invasion. The archaeological and historical debate has conflated both issues and explanations for the former have been falsely viewed as direct rivals of explanations of the latter. But motivation and effectiveness naturally have different sources. The motivation of the potential invaders is shaped by the value h that the challenger assigns to the asset v, as studied in the extension in Section XXXX. The parameter h captures a number of contextual factors affecting the voracity of the challenger for the output of its civilized neighbor, including certainly a famine induced by climate change.

The effectiveness of barbarian arms against the civilized centers is captured by the parameter kappa_c in the extension developed in Section XXXX. While changes in h and kappa capture substantively different forces, as shown in our extension they are mathematically equivalent in that both affect the aggressiveness of the challenger in the margin. A world with a more aggressive challenger is tougher on the incumbent, and it is possible that as a result of a change in the challenger's aggressiveness a polity may go from enjoying both peace and security to enjoying only one of those or neither. A sufficiently large change in any of those parameters h and kappa_c could take a polity to a situation where it faces small odds of prevailing against its attacker.

Specifically, the historical victory of the "Sea Peoples" over the cities and kingdoms of the Ancient Near East involved a shift from the order-prosperity quadrant of our map (R1, or good parts of R2) to the conflict-stagnation quadrant (R3), as a result of positive shocks to the value of the economic output to the challengers (an increase in h, in turn an effect of a climate change) or in the technology of the means of destruction (an increase in kappa_c).

We can use the model to delve deeper into the diverging fates of the different regions that suffered the attacks of the Sea Peoples at the end of the Bronze Age. The extremes of that contrast are Egypt, which managed to repel the invasion, and cities near the Mediterranean coast of the Levant, like Ugarit, for which invasion resulted in irreversible destruction.

Ugarit is a model case of Bronze Age collapse because, in addition to its pre-invasion political complexity and economic prosperity, the archeological excavation found clay tablets that survived the destruction to provide the most dramatic textual evidence on the threat of the Sea Peoples as well as on the efforts to prepare against them, which eventually proved completely futile. In the tablets, the king of Ugarit makes desperate requests to his Anatolian overlord, who was using Ugarit's maritime fleet to defend other sections of the Hittite empire. The tablets reveal that the attack was formidable and Ugarit was almost defenseless.[9] Hence,

[9] Ammurapi, the King of Ugarit, makes a desperate plea to the Hittite overloard, whom he addresses as his "father:" "My father, behold, the enemy's ships came (here); my cities(?) were burned, and they did evil things in my country. Does not my father know that all my troops and chariots(?) are in the Land of Hatti, and all my ships are in the Land of Lukka?...Thus, the country is abandoned to itself. May my father know it: the seven ships of the enemy that came here inflicted much damage upon us". Letter RS 18.147 in Jean Nougaryol et al. 1968. Ugaritica V, 24: 87–90.

the eventual destruction. In terms of our model, Ugarit's vulnerability at the time of the invasions can be interpreted as either an initial location within R1 that was close to the vortex, and thereby not too far away from R3, or within a narrow strip along the R2/R3 border on the side of R2. When the shocks that prompted the invasion occurred—which, as already seen, involved a redrawing of the borders of the regions—the effect was to push Ugarit deep into R3. In the new location, Ugarit faced the prospects of attacks that were too strong for the city to resist. In the model, a deep position in R3 (i.e., close to the rho axis) entails a very low probability that the incumbent will prevail. In fact, for Ugarit, foreign attack resulted in extinction.

Egypt was also a victim of Barbarian attacks, and repeatedly so, but the outcome was very different as Egypt managed to resist and survive. Since the end of the Second Intermediate Period, Egypt had developed a military force of genuinely imperial strength, including a highly professional army and a formidable fleet of ships with the most advanced equipment. Before the shock, Egypt was located deep in R1 so that the worst effect of the shock could have been to relocate Egypt in a relatively safe neighborhood of R2. Egypt became susceptible to challenges, but it could prevail in the battlefield with high probability. Pictorial inscriptions on the walls of the Karnak Temple attest the threats posed by the Sea Peoples at roughly the same time they invaded the Levant. But, in contrast to Ugarit's tablets, the Egyptian inscriptions actually honor king Merneptah's success in subduing the invaders. The contrasting cases of Ugarit and Egypt correspond to the points A and B in Figure XXX.

Endogenous military capacity and the transitions to order and prosperity

Setting

Let us now introduce a period zero, before the periods 1 and 2 that we have analyzed so far. This allows us to endogenize military capacity. We will model this by allowing the incumbent to spend resources in one period to increase its military effectiveness in the next period. Since the challenger will never fight in period 2, the incumbent will never spend in expanding military capacity in period 1. Thus, the decision to augment military capacity will be relevant only in period 0. We postulate that in period 0 the incumbent has a military capacity κ_0, and can spend an amount m_0 that will take military capacity in the next period to $\kappa_1 = \kappa_0 + \gamma m_0$. To make things interesting, we assume κ_0, ρ are such that if things were left unchanged, in period 1 the incumbent would find himself in region $R3$, which means he can expect disorder and stagnation. In particular, we impose the following,

Assumption : $\rho\kappa_0 < 4$ and $\kappa_0 < 2$.

All other aspects of the interaction between challenger and incumbent remain as before.

Timing

In period 0, the incumbent starts by selecting m_0. Then, in each period the incumbent selects a_t and i_t.[10] After observing (a_t, i_t) the challenger selects b_t. If $b_t = 0$, the players retain their positions in the next period. If $b_t > 0$, then there is a war at the end of period t. The winner of the war becomes the incumbent in the next period, and faces a new challenger then.

Payoffs

The fact that there is a new type of expenditure changes the incumbent's budget constraint to $v_0 - m_0 - \frac{a_0}{\kappa_0} - i_0 \geq 0$. And the fact that there is an extra period now implies that a zero arming decision by the challenger in period 1 could open the challenger to vulnerability. So in this three-period model an additional assumption is that the challenger cannot be eliminated.[11] This matches the historical cases of settlers dealing with nomadic raiders, who have vast steppes on which to run away from the forces of the civilized, settled, center.

As before, we solve the model through backward induction. The solution for periods 1 and 2 is given by our analysis in the previous section. That analysis tells us the expected payoff for being an incumbent in period 1 is given by,

$$V_1(i_0, m_0) = (v_0 + \rho i_0) \times \begin{cases} \frac{(\kappa_0+\gamma m_0)(1+\rho)}{\kappa_0+\gamma m_0+\rho} & (\kappa_0 + \gamma m_0,\ \rho) \in \mathbf{R1} \\ \sqrt{\frac{(\kappa_0+\gamma m_0)}{\rho}} \frac{(1+\rho)}{2} & (\kappa_0 + \gamma m_0,\ \rho) \in \mathbf{R2} \\ \left(1 + \frac{\kappa_0+\gamma m_0}{4}\right) & (\kappa_0 + \gamma m_0,\ \rho) \in \mathbf{R3} \\ \left(2 - \frac{1}{\kappa_0+\gamma m_0}\right) & (\kappa_0 + \gamma m_0,\ \rho) \in \mathbf{R4} \end{cases} \equiv (v_0 + \rho i_0) S(m_0)$$

Given this continuation value, we can solve for decisions in period 0. After the incumbent has selected m_0, a_0 and i_0, the challenger decides whether to arm himself. Using the same logic as in the previous section, we see that the challenger's best response function is given by,

[10] The assumption that m_0 is decided before a_0 and i_0 is immaterial and will just simplify the exposition. It is equivalent to assume that the incumbent selects all three variables simultaneously. What is of course important is that the incumbent makes his choices before the challenger.

[11] If the challenger can be eliminated when selecting zero arming, then it would not be an equilibrium for the challenger to desist from arming itself.

$$b_0(a_0, m_0, i_0) = \begin{cases} \sqrt{a_0 V_1(i_0, m_0)} - a_0 & if\ a_0 < V_1(i_0, m_0) \\ 0 & if\ a_0 \geq V_1(i_0, m_0) \end{cases}$$

This notation embeds the four regions over which $V_1(i_0, m_0)$ is defined into the calculus of the challenger. Given this best response function, the incumbent has to choose a_0, i_0 after it chose m_0 such that she maximizes her expected utility.

The incumbent maximizes,

$$\max_{a_0, i_0 \geq 0} v_0 - m_0 - \frac{a_0}{\kappa_0} - i_0 + \frac{a_0}{a_0 + b_0(a_0, i_0, m_0)} V_1(i_0, m_0)$$

subject to

$$\begin{aligned} v_0 - m_0 - \frac{a_0}{\kappa_0} - i_0 &\geq 0\ (BC) \\ (v_0 + \rho i_0) S(m_0) - a_0 &\geq 0\ (ND) \\ a_0 &\geq 0 \\ i_0 &\geq 0 \end{aligned}$$

Notice this problem is similar to the one with two periods in the previous section, except now the continuation value depends explicitly on m_0 (which is fixed at this stage) through $S(m_0)$. The objective function is differentiable in a_0 and i_0. As before, the first order and complementary slackness conditions that characterize the optimum are given by

$$a_0 : \frac{1}{2}\sqrt{\frac{(v_0 + \rho i_0)\, S(m_0)}{a_0}} - \frac{1}{\kappa_0} - \frac{\lambda_{BC}}{\kappa_0} - \lambda_{ND} + \lambda_a = 0 \tag{12}$$

$$i_0 : \frac{\rho}{2}\sqrt{\frac{a_0}{(v_0 + \rho i_0)\, S(m_0)}} - 1 - \lambda_{BC} + \lambda_{ND} \rho S(m_0) + \lambda_i = 0 \tag{13}$$

$$\lambda_{BC}(v_0 - m_0 - \frac{a_0}{\kappa_0} - i_0) = 0,\ \lambda_{ND}\left((v_0 + \rho i_0) S(m_0) - a_0\right) = 0,\ \lambda_a a_0 = 0,\ \lambda_i i_0 = 0 \tag{14}$$

As before, $\lambda_{BC}, \lambda_{ND}$ are the Lagrange multipliers for the budget constraint and non-deterrence constraints, and λ_a, λ_i are the multipliers for the non-negativity constraints for the control variables. Also as before the infinite marginal utility of a_0 at zero implies $a_0 > 0$ and $\lambda_a = 0$, so there are in principle eight possible cases depending on whether the remaining three Lagrange multipliers are positive or zero. The following Lemma shows that, given our Assumption 2.2 there are only two feasible cases in period 0.

Lemma 1 *If Assumption 2.2 holds, then in period 0 the incumbent chooses:*

i) $i_0 = 0$ *and* $a_0 = \frac{\kappa_0^2}{4} v_0 S(m_0)$ *when* $\frac{v_0 S(m_0)}{(v_0 - m_0)} < \frac{4}{\kappa_0}$*; or*

ii) $i_0 = 0$ *and* $a_0 = \kappa_0(v_0 - m_0)$ *when* $\frac{v_0 S(m_0)}{(v_0 - m_0)} \geq \frac{4}{\kappa_0}$.

Proof: See Appendix.

Lemma 1 reveals that no productive investment is carried away on period 0 and that the army size depends on the value of m_0.With this result, we are now equipped to study the incentives of the incumbent to make changes in military capacity m_0. We can trace how those changes will affect period 0 investment and army decisions for both incumbent and challenger and, consequently, future investment, army sizes, peace and prosperity outcomes.

2.3 Endogenous military capacity

In our case of interest, for any value of m_0 we are able to compute the incumbent's present expected utility, given our result in Lemma 1. The effect of m_0 on the incumbent's utility depends on the iniitial conditions in period 0. If the maximum utility comes from extremely low m_0 then the incumbent will be in the zone with no growth and war (**R3**) in period 1. On the contrary, if the optimal m_0 is extremely high, investment and peace will obtain in period 1. Notice, however, that the path to prosperity (via investments in m_0) depends on ρ–the return on productive investment. In particular, when $\rho < 2$, the path to peace and prosperity requires going from **R3** to **R1** through **R4**, and when $\rho \geq 2$, it requires going from **R3** to **R1** through **R2** (see Figure **??**).We analyze these cases in turn.

Case $\rho < 2$

In this case, the incumbent starts at **R3** (conflict and stagnation) and might either stay at **R3**, move to **R4** (attain order) or evolve to **R1** (attaining both order and prosperity) depending on initial income measured in terms of military capacity units.

Proposition 3 *Under Assumption 2.2 and provided that* $\rho < 2$, *there exist cutoffs* τ_L, τ_M *and* τ_H, $\tau_L < \tau_M \leq \tau_H$ *such that*

1. *If* $\gamma v_0 < \tau_L$, *the polity stays in R3 (stagnation without order);*
2. *If* $\tau_H < \gamma v_0$, *the polity moves to R1 (attains order and prosperity); and*
3. *If* $\tau_M < \gamma v_0 < \tau_H$, *the polity moves to R4 (attains order without prosperity)*

Proof: See Appendix.

The expression γv_0 captures the extent to which initial income can be used to accumulate military capacity. This proposition tells us that, the initial military capacity κ_0 and the productivity of investment ρ, the transitions followed by the polity will be very different depending on the initial level of prosperity (in terms of ability to purchase military capacity) γv_0. If γv_0 is very low, the polity will remain trapped without order or prosperity. If γv_0 lies in an intermediate region, the polity will move into **R4** where it will attain peace, but given the low $\rho < 2$ it will not be able to grow. The reason is that even though it attains a higher military capacity κ_1 in the next period, which gives giving the incumbent the ability to fend off attacks at a lower cost, the benefit from consumption will still be higher than the present value from investing. If γv_0 is very high, however, the subsequent military capacity κ_1 will allow the incumbent to free resources for both a deterrent army and a large-scale investment at **R1**.

Case $\rho \geq 2$

In this case, the incumbent starts at **R3** (stagnation without order) and might either stay at **R3**, move to **R2** (prosperity without order) or evolve to **R1** (order and prosperity). As before, the following result establishes the conditions on the return of the productive investment and on the productivity of the military capacity investment such that this transitions obtain.

Proposition 4 *Under Assumption 2.2 and provided that $\rho \geq 2$, there exist cutoffs σ_L, σ_M and σ_H, $\sigma_L \leq \sigma_M \leq \sigma_H$ such that*

1. *If $\gamma v_0 < \sigma_L$, the polity stays in R3 (stagnation and conflict);*
2. *If $\sigma_H < \gamma v_0$, the polity moves to R1 (attains order and prosperity); and*
3. *If $\sigma_M < \gamma v_0 < \sigma_H$, the polity moves to R2 (attains prosperity without order)*

Proof: See Appendix.

Thus, the two cases $\rho < 2$ and $\rho \geq 2$ yield a picture with a commonality and a difference. The commonality is that if initial income (measured in terms of purchasing power of military capacity) γv_0 is small, the polity will remain stagnant and violent while if γv_0 is large enough the polity will have sufficient resources to conquer order and prosperity. However, if γv_0 is intermediate transition followed by the polity will be different. For $\rho \geq 2$ the polity will attain prosperity in period 1 but remain violent, while for $\rho < 2$ the polity will attain order in period 1 but remain poor. To summarize, while large enough initial income (or cheap enough military capacity) guarantees order and prosperity through sufficient accumulation of military capacity, intermediate levels will allow to attain either order *or* prosperity. Which one is attained depends on the value of ρ. Thus, the emerging picture is one that situates the Hobbesian argument in very specific place in the process of attainment of order and prosperity. Not only is order not always a precondition of prosperity (in $R2$ we can have without the other) but it is initial income (as captured by v_0) which allows the polity to augment the military capacity to the level that could ensure order.

3 Historical Illustration: Sumeria and the origin of civilization

The Fertile Crescent in Southwest Asia is the cradle of civilization. It was the source of the first substantial volumes of economic surplus in human evolution, in turn the result of a major innovation: domestication of plants and animals for food production. The Fertile Crescent was a political pioneer as well. The rudiments of large-scale political organization emerged in Southern Mesopotamia to form the pristine city-states of Sumer and ultimately shape the first major civilization.

Like in Egypt, in Mesopotamia it was a fertile river valley, exceptionally endowed for alluvial agriculture, that was the key for economic prosperity. The twin rivers Tigris and Euphrates flooded the land and replenished nutrients by spreading silt. Also like in Egypt, the blessings of nature required systematic human effort to produce economic results. Southern Mesopotamians made massive investments in the creation of the proper irrigation infrastructure. The investments were made because of extraordinary returns. According to Mann, "If [the alluvium] can be diverted onto a broad area of existing land, then much higher crop yields can be expected. This is the significance of irrigation in the ancient world: the spreading of water and silt over the land. Rain-watered soils gave lower yields" (1986: 78). Liverani offers an idea of the increase in yields that could be obtained through judicious investments: "The agricultural production of barley underwent a notable, possibly tenfold, increase thanks to the construction of water reservoirs and irrigation canals, of long fields adjacent to the canals watered by them, and thanks to the use of the plow, of animal power, of carts, of threshing sledges, of clay sickles, and of improved storage facilities" (2005, p. 5).

These high returns to productive investment help place Sumeria in the parameter space of our model as a case of high rho. What about the other parameter, the productivity of investments in military defense, kappa?

In contrast to Egypt, geography did not afford the Sumerians natural protection against attacks from outsiders. On the contrary, the natural landscape exposed Sumeria to numerous threats. As Bradford puts it, “Their neighbors to the west, the Amorites, nomads of the desert, infiltrated Mesopotamia... The neighbors to the east, who dwelled in the mountains, were the Gutians and the Elamites. The Gutians and, to a lesser extent, the Elamites considered Sumer and Akkad a treasurehouse to be raided” (1993: 4). (Finer (1997: 102) also emphasized the porousness of frontiers.)

In terms of our model, the vulnerability of Sumeria to invaders means that the productivity of defensive effort was low (low kappa). Given a low kappa, Sumeria’s trajectory must have begun in the conflict-stagnation region, R3. But if output was so insecure, how could the first human civilization emerge at all? That is, how did Sumerians solve the problem of protecting surpluses from nomadic raiders and encouraging investments in productive infrastructure?

The extended model featuring endogenous military capability provides an answer. Our proposition #XX states that a polity that is initially in R3 due to a low military capability kappa_0, may invest in military capability in order to attain sufficient security against the challenger. A key condition for this investment to be undertaken is that the polity has enough initial income v. The archaeological record suggests Sumeria was well placed to meet that requirement. The availability of alluvial agriculture combined with an unparalleled initial endowment of plant and animal domesticates furnished the entire Fertile Crescent with exceptional advantages in food production. It is well known that due to altitude and climactic variation, the Fertile Crescent hosted a wide variety of plants with high potential for food production. The region had plants that already had high yields of edible content in the wild, a high proportion of hermaphroditic plants that were more amenable to experimentation and selection based on yield, and a number of crops with high protein content. These facts are well summarized by Diamond (1997: Ch. 8), who notes that all eight founder crops in the Neolithic were present in the area (these would be the wild ancestors of einkorn, emmer wheat, flax, lentil, chick pea, pea, bitter vetch and barley). In addition, out of the five most important domesticated animals, four were available in the Fertile Crescent, namely pigs, cows, sheep and goats. Given the natural advantages, Diamond claims that “any attempt to understand the origins of the modern world must come to grips with the question why the Fertile Crescent’s domesticate plants and animals gave it such a potent head start” (1997: 135). In terms of our model, this combination of initial advantages can be captured by a high v.

What is tricky about the role of the Fertile Crescent's initial advantages is that a high v can encourage predation by outside challengers. However, a high v could also help finance the investments in defense that were needed to escape the conflict-stagnation trap characterizing region R3. It is by no means obvious that the "defense-financing" force should dominate the predation force. This is a key tension investigated by our model, and the model makes an unambiguous prediction: for v low enough, no escape from R3 is possible. For v high enough, the incentive to finance defensive capabilities dominates. Under such conditions, the investments in defense are made, and they bring enough security so as to incentivize productive investments and economic progress.

Now what is the evidence of military capabilities that grew endogenously in Sumeria? The archaeological record offers striking evidence of large and generalized investments to improve defense in the form of protective perimeter walls, which made Sumerian cities large-scale fortifications. The figure below includes illustrations of a number of Sumerian cities. All of them had walls. In fact, virtually every city in ancient history had walls. x In the case of Sumeria (and virtually every other civilization in history) the walls were the endogenous, artificial substitute for the missing natural protection.

According to van de Mieroop's study of Mesopotamian cities, "The inner cities were also clearly distinguished by their defensive walls. Perhaps the presence of walls was the main characteristic of a city in the eyes of an ancient Mesopotamian: all representations of cities prominently display walls, many kings boast of their building or repairing city walls, and even literary works sing their praise. A city without a wall might thus not have been conceivable... The typical image of a city consisted of one or more rings of fortification walls with numerous towers at regular intervals, which seem to indicate a walled citadel and one or more town walls... The ubiquitous emphasis on the walls in iconographic material reveals the Mesopotamian concept that they were a crucial characteristic of the city. That concept is also reflected in the numerous royal building inscriptions commemorating the construction or repair of city walls."

The archaeological record substantiates not only the generalized presence of military investments in rising city-states, but also their costliness, which would have been prohibitive to societies with low initial productive capacity. Both walls and the often complementary moats have been estimated to have required large investments of labor. When commenting on the Babylonian city of Dur-Jakin, Van de Mieroop mentions estimates that the construction of the city's moat would have taken ten thousand men working for three and a half months. (Van de Mieroop, p. 76).

We have just explained how the model highlights the reason why the initial advantages of the Fertile Crescent would have been a necessary, if at first seeminly paradoxical, condition for the rise of civilization in a challenger-ridden area. The high initial v helped finance defensive capabilities as societies got ready to expand their surplus. But the model also highlights why those advantages would have not been sufficient. Indeed, the pristine rise of civilization occurred in Southern, alluvial, Mesopotamia, and not in the Northern, rain-fed areas. The differentiating factor between the two, according to our model, is that the Southern area boasted the high rho.

At a very general level, our model emphasizes the role of investment, an intertemporal bet, in the rise of the first civilizations. This matches the emphasis by Liverani (2005), according to whom "The technological improvements alone, however, could generate no "revolution" at all if the food-producers had devoted the entire surplus to their own consumption" since it was necessary to direct resources both to productive and defensive infrastructures..

4 Applications to modern states

Argentina and wool

Postcolonial Africa.

State-building efforts

The remarks in Section XXXX help think about a crucial problem in state-building. The consensus in the military community is that counterinsurgency and reconstruction of broken states require substantial development initiatives, in addition to reinstating a military and policy capacity.[12] What should be the balance between the two? If development initiatives are associated with improvements in ρ, and military assistance is associated with improvements in κ, these remarks tell us that improvements in κ alone can yield state consolidation and growth, but the cheapest way might be to add improvements in ρ once a minimum domestic military capacity has been established. The reason is that once the polity is in $R4$, the shortest route to $R1$ is to increase ρ. This follows from the fact that the frontier between these two regions has slope $\frac{-1}{(\kappa-1)^2}$ which is smaller than -1 whenever $\kappa > 2$.

[12] A well known example of this thinking is the US Army manual on counterinsurgency (Headquarters of the Army 2006).

5 Conclusion

We present a model to investigate the dynamics of productive and military capacities in a polity where an incumbent seeks to consolidate power and grow the economy. The model helps evaluate claims about the relative role of productive vs. military capacities in the attainment of order and prosperity that are central to classic theories of state formation. One can conceive of four distinct combinations of order, seen as the peace that follows the monopolization of violence, and prosperity, seen as the ability to increase income over time. An interesting question is whether all four regimes can take place as equilibrium outcomes, or whether the logic of conflict and accumulation incentives precludes some of them. A basic model with exogenous military capacities shows that all four regimes are possible, preventing simple characterizations of order or prosperity being necessary or sufficient conditions for one another. These regimes match various historical experiences of societies that attained neither, one, or both objectives. In that basic model, higher initial income does not affect the resulting regime the polity is in, but tends to exacerbate conflict. In addition, the basic model helps consider how exogenous shocks to productive and military capacities–as might be created by foreign assistance in the process of state-building–might affect the ability of a society to attain order and prosperity. The take-away is that a balance between productive and military aid might have to be maintained to prevent adverse effects.

We then expand the model to consider endogenous military capacity. Income windfalls, which exacerbated conflict in the basic model, can now be the key to transitioning into order and prosperity. The possibility of accumulating military capacity helps create the conditions where productive investments can be made without triggering predatory challenges. This result can rationalize historical experiences where a temporary economic boom allows the state to consolidate its power and usher in a phase of more sustained growth. This result on the pivotal role of military capacity contributes to the demanding enterprise of discerning how economic shocks can hinder or help state formation and political stability.

6 Appendix

Proof of 1: Given that $\lambda_a = 0$, we have eight possible cases given by whether the three remaining Lagrange multipliers $\lambda_{BC}, \lambda_{DC}$, and λ_i are zero or positive. We analyze each one of them.

1. **Case** $\lambda_{BC} > 0$ **(BC binds)**, $\lambda_{DC} > 0$ **(DC binds, consolidation)**,**and** $\lambda_i = 0$

$(i_1 > 0)$

It proves useful to determine for which subset $\{(\kappa_1, \rho, v_1) | \kappa_1, \rho, v_1 \geq 0\}$ all the conditions (9), (10) and (11) hold. This is

$$
\begin{aligned}
\frac{1}{2}\sqrt{1} - \frac{1}{\kappa_1} - \frac{\lambda_{BC}}{\kappa_1} - \lambda_{DC} &= 0 \\
\frac{\rho}{2}\sqrt{1} - 1 - \lambda_{BC} + \lambda_{DC}\rho &= 0 \\
v_1 - a_1 + \rho i_1 &= 0 \\
v_1 - \frac{a_1}{\kappa_1} - i_1 &= 0
\end{aligned}
$$

In this case, investment and army are given by

$$
\begin{aligned}
i_1 &= v_1 \frac{(\kappa_1 - 1)}{(\kappa_1 + \rho)} \\
a_1 &= v_1 \frac{\kappa_1(1+\rho)}{(\kappa_1 + \rho)}
\end{aligned}
\tag{15}
$$

As a result $\lambda_i = 0$ (or $i_1 > 0$) is supported by $\kappa_1 > 1$. After some algebra (using the first two equations) we find that $\lambda_{DC} > 0$ if and only if $\kappa_1 > \rho$ and $\lambda_{BC} > 0$ if and only if $\rho > \kappa_1/(\kappa_1 - 1)$. Therefore the parameter set such that this is the solution to the incumbent's problem in period 1 is given by

$$
\mathbf{R1} = \left\{ (\kappa_1, \rho, v_1) \in \mathbb{R}^3_+ | \kappa_1 > \rho,\ \rho > \kappa_1/(\kappa_1 - 1)\, and\, \kappa_1 > 1 \right\},
$$

and in this area there is investment and deterrence of the challenger, yielding consolidation.

The expected utility in period 1 is in each case easily computed by substituting the solutions into the Lagrangian. In this first case expected utility is,

$$
E[U_1] = v_1 \frac{\kappa_1(1+\rho)}{(\kappa_1 + \rho)}.
$$

2. Case $\lambda_{BC} > 0$ (BC binds), $\lambda_{DC} = 0$ (DC does not bind, conflict),and $\lambda_i = 0$ $(i > 0)$

Again we check the first order and complementary slackness conditions to see for which parameter set this case contains the solution. The relevant conditions are,

$$
\begin{aligned}
\frac{1}{2}\sqrt{\frac{v_1+\rho i_1}{a_1}}-\frac{1}{\kappa_1}-\frac{\lambda_{BC}}{\kappa_1} &= 0\\
\frac{\rho}{2}\sqrt{\frac{a_1}{v_1+\rho i_1}}-1-\lambda_{BC} &= 0\\
v_1-\frac{a_1}{\kappa_1}-i_1 &= 0
\end{aligned}
$$

Investment and army solutions are respectively given by,

$$
\begin{aligned}
i_1 &= \frac{v_1}{2}\left(1-\frac{1}{\rho}\right)\\
a_1 &= \frac{\kappa_1 v_1}{2}\left(1+\frac{1}{\rho}\right).
\end{aligned}
$$

This solution is consistent with $\lambda_{DC}=0$ (satisfies (DC) with strict inequality) and $\lambda_i=0$ if and only if $\rho>\kappa_1$ and $\rho\geq 1$. It is consistent with $\lambda_{BC}>0$ if and only if $\rho\geq 4/\kappa_1$ (this comes from checking the conditions such that $\lambda_{BC}>0$ in the first two equations). As a result, the parameter set for which these are the optimal army and investment is given by

$$
\mathbf{R2}=\left\{(\kappa_1,\rho,v_1)\in\mathbb{R}^3_+ | \rho>\kappa_1,\ \rho\geq 4/\kappa_1\, and\, \rho\geq 1\right\}
$$

Expected utility for the incumbent in this case is,

$$
E[U_1]=\frac{v_1}{2}\left(1+\frac{1}{\rho}\right)\sqrt{\rho\kappa_1}
$$

3. Case $\lambda_{BC}=0$ **(BC does not bind)**, $\lambda_{DC}=0$ **(DC does not bind, conflict)**,**and** $\lambda_i=0$ **(**$i_1>0$**)**

Non-generic, in the sense that it is consistent only over a subset of the space (ρ,κ_1,v_1) that has measure zero. Proof to be included.

4. Case $\lambda_{BC}=0$ **(BC does not bind)**, $\lambda_{DC}>0$ **(DC binds, consolidation)**,**and** $\lambda_i=0$ **(**$i_1>0$**)**

Non-generic. Proof to be included.

5. Case $\lambda_{BC}>0$ **(BC binds)**, $\lambda_{DC}>0$ **(DC binds, consolidation)**,**and** $\lambda_i>0$ **(**$i_1=0$**)**

Non-generic. Proof to be included.

6. Case $\lambda_{BC}>0$ **(BC binds)**, $\lambda_{DC}=0$ **(DC does not bind, conflict)**,**and** $\lambda_i>0$ **(**$i_1=0$**)**

Not feasible. This case yields $a_1 = v_1\kappa_1$ and $i_1 = 0$. This case is consistent if and only if $\kappa_1 > 4$ (which is the case with $\lambda_{BC} > 0$). However, for $\kappa_1 > 4$, the (DC) constraint is violated ($v_1 - v_1\kappa_1 > 0$ iff $\kappa_1 < 1$). That is, there is no profile of parameter values such that the optimum satisfies the conditions in this case.

7. Case $\lambda_{BC} = 0$ **(BC does not bind)**, $\lambda_{DC} = 0$ **(DC does not bind, conflict), and** $\lambda_i > 0$ **(**$i_1 = 0$**)**

In this case $a_1 = v_1\kappa_1^2/4$ and $i_1 = 0$. This solution is consistent with $\lambda_{BC} = 0$ and $\lambda_{DC} = 0$ if and only if $\kappa_1 \leq 2$. Also for $\lambda_i > 0$ we need $1 - \rho\kappa_1/4 > 0$ (from the FOC of i_1). Thus, this holds for any triple $(\rho, \kappa_1, v_1) \in \mathbb{R}^3_+$ such that $\kappa_1 \leq 2$ and $\rho < 4/\kappa_1$. In other words, the parameter set for which this region contains the solution to the incumbent's problem is

$$\mathbf{R3} = \left\{(\kappa_1, \rho, v_1) \in \mathbb{R}^3_+ | 2 \geq \kappa_1 \, and \, \rho < 4/\kappa_1\right\}.$$

Expected utility in this case is given by,

$$E[U_1] = v_1\left(1 + \frac{\kappa_1}{4}\right)$$

8. Case $\lambda_{BC} = 0$ **(BC does not bind)**, $\lambda_{DC} > 0$ **(DC binds, consolidation),and** $\lambda_i > 0$ **(**$i_1 = 0$**)**

In this case the system of conditions is given by

$$\begin{aligned}
\frac{1}{2}\sqrt{1} - \frac{1}{\kappa_1} - \lambda_{DC} &= 0 \\
\frac{\rho}{2}\sqrt{1} - 1 + \lambda_{DC}\rho + \lambda_i &= 0 \\
v_1 - a_1 + \rho i_1 &= 0.
\end{aligned}$$

Investment $i_1 = 0$ and therefore from (DC) we get $a_1 = v_1$. For this to be consistent with $\lambda_{DC} > 0$, we must have from the first equation that $\kappa_1 > 2$, and to be consistent with $\lambda_i > 0$ we need $\rho < \kappa_1/(\kappa_1 - 1)$. Therefore the region such that this represents the optimal solution is given by

$$\mathbf{R4} = \left\{(\kappa_1, \rho, v_1) \in \mathbb{R}^3_+ | \kappa_1 > 2, \, \rho < \kappa_1/(\kappa_1 - 1)\right\}.$$

The expected utility in this case is,

$$E[U_1] = v_1\left(2 - \frac{1}{\kappa_1}\right).$$

■

Proof of Proposition 2: That a_1 increases in v_1 in all regimes follows directly from inspection of the solution for a_1 in each regime in Proposition 1. To see that whenever positive b_1 also increases in v_1, take the value of b_1 from the best response expression (4), and substitute in the values of i_1, a_1 in regions R2 and R3. This yields respectively,

$$\begin{aligned} b_{1,R2} &= v_1 \left\{ \sqrt{\frac{\kappa_1}{2}\left(1+\frac{1}{\rho}\right)\left(1+\rho\frac{\kappa_1-1}{\kappa_1+\rho}\right)} - \frac{\kappa_1}{2}\left(1+\frac{1}{\rho}\right) \right\} \\ b_{1,R3} &= v_1 \left(\frac{\kappa_1}{2}\right)\left\{1-\frac{\kappa_1}{2}\right\} \end{aligned}$$

which are both positive and increasing in v_1.

Proof of Lemma 1:

Case $\lambda_{BC} = 0$, $\lambda_{ND} = 0$, **and** $\lambda_i > 0$

FOC

$$\frac{\partial \mathbf{L}}{\partial a_0} = \frac{1}{2}\sqrt{\frac{v_0 S(m_0)}{a_0}} - \frac{1}{\kappa_0} = 0 \tag{16}$$

$$\frac{\partial \mathbf{L}}{\partial i_0} = \frac{\rho}{2}\sqrt{\frac{a_0}{v_0 S(m_0)}} - 1 + \lambda_i = 0 \tag{17}$$

This implies:

$$i_0 = 0$$

$$a_0 = \frac{\kappa_0^2}{4} v_0 S(m_0)$$

$$\lambda_i = 1 - \frac{\rho\kappa_0}{4}$$

The necessary and sufficient conditions for this case to hold are

$$\lambda_{BC} = 0 \iff \frac{v_0 S(m_0)}{c_0(v_0 - m_0)} < \frac{4}{c_0^2}$$

$$\lambda_{ND} = 0 \iff 1 > \frac{c_0^2}{4} \iff c_0 < 2$$

$$\lambda_i > 0 \iff 1 - \frac{\beta c_0}{4} > 0 \iff \beta < \frac{4}{c_0}$$

The first one holds for values of m_0 low enough and the second and third hold by assumption 2.2.

Case $\lambda_{BC} > 0$, $\lambda_{ND} = 0$,**and** $\lambda_i > 0$

$$\frac{\partial \mathbf{L}}{\partial a_0} = \frac{1}{2}\sqrt{\frac{v_0 S(m_0)}{a_0}} - \frac{1}{\kappa_0} - \frac{\lambda_{BC}}{\kappa_0} = 0 \tag{18}$$

$$\frac{\partial \mathbf{L}}{\partial i_0} = \frac{\rho}{2}\sqrt{\frac{a_0}{v_0 S(m_0)}} - 1 - \lambda_{BC} + \lambda_i = 0 \tag{19}$$

$\lambda_{BC} > 0$ implies $\kappa_0 (v_0 - m_0) = a_0$ which is always true.

From FOC, we obtain

$$\lambda_{BC} = \frac{\kappa_0}{2}\sqrt{\frac{v_0 S(m_0)}{a_0}} - 1$$

$$\lambda_i = \frac{\kappa_0}{2}\sqrt{\frac{v_0 S(m_0)}{a_0}} - \frac{\rho}{2}\sqrt{\frac{a_0}{v_0 S(m_0)}}$$

As before, the conditions on the parameters for this to be a solution are

$$\lambda_{BC} = \frac{c_0}{2}\sqrt{\frac{v_0 S(m_0)}{a_0}} - 1 > 0 \iff \frac{v_0 S(m_0)}{c_0(v_0 - m_0)} > \frac{4}{c_0^2}$$

$$\lambda_{ND} = 0 \iff \frac{v_0 S(m_0)}{c_0(v_0 - m_0)} > 1$$

$$\lambda_i = \frac{c_0}{2}\sqrt{\frac{v_0 S(m_0)}{c_0(v_0 - m_0)}} - \frac{\beta}{2}\sqrt{\frac{c_0(v_0 - m_0)}{v_0 S(m_0)}} > 0 \iff \frac{v_0 S(m_0)}{\beta(v_0 - m_0)} > 1$$

By assumption 2.2 the first case obtains when $\frac{v_0 S(m_0)}{(v_0 - m_0)} < \frac{4}{\kappa_0}$ and the second case when $\frac{v_0 S(m_0)}{(v_0 - m_0)} \geq \frac{4}{\kappa_0}$ .The proof that the other cases are infeasible (that is, the conditions on the parameters that support them do not hold under assumption 2.2) is tedious but straightforward.■

Proof of Proposition 3: Recall that under Assumption 2.2 there are only two feasible cases:

Case $\lambda_{BC} = 0$, $\lambda_{ND} = 0$, **and** $\lambda_i > 0$

$$\lambda_{BC} = 0 \iff \frac{v_0 S(m_0)}{(v_0 - m_0)} < \frac{4}{c_0}$$

$$\lambda_{ND} = 0 \iff c_0 < 2$$

$$\lambda_i > 0 \iff \beta < \frac{4}{c_0}$$

Expected Utility

$$EU = v_0 - m_0 + \frac{\kappa_0}{4} v_0 S(m_0)$$

Case $\lambda_{BC} > 0$, $\lambda_{ND} = 0$,**and** $\lambda_i > 0$

$$\lambda_{BC} > 0 \iff \frac{v_0 S(m_0)}{(v_0 - m_0)} > \frac{4}{c_0}$$

$$\lambda_{ND} = 0 \iff \frac{v_0 S(m_0)}{(v_0 - m_0)} > c_0$$

$$\lambda_i > 0 \iff \frac{v_0 S(m_0)}{(v_0 - m_0)} > \beta$$

Expected utility:

$$EU = \sqrt{\kappa_0 \left(v_0 - m_0\right) v_0 S(m_0)}$$

From these two sets of the parameter space, we can compute the values of EU in period $t = 0$ for each m_0 fixing all the other parameters.

Let us call $\bar{m}$ the value of m_0 that satisfies this equation: $\frac{v_0 S(\bar{m})}{\kappa_0(v_0 - \bar{m})} = \frac{4}{\kappa_0^2}$. $\bar{m}$ is the value of m_0 at which regimes change in period $t = 0$.

Let us call $m_{R3|R4}$ and $m_{R4|R1}$ the values of m_0 such that regimes change in period $t = 1$: $m_{R3|R4} = \frac{2-\kappa_0}{\gamma}$ and $m_{R4|R1} = \frac{1}{\gamma}\left(\frac{\rho}{\rho-1} - \kappa_0\right)$, $m_{R3|R4} < m_{R4|R1}$. $\bar{m}$ depends on whether it is above or below $m_{R3|R4}$ or $m_{R4|R1}$, because $\bar{m}$ is an implicit function of $S()$.

Before proving Proposition **3** we need a technical result. The following Lemma establishes the conditions of the parameters that determines the value of $\bar{m}$ relative to $m_{R3|R4}$ and $m_{R4|R1}$.

Lemma 2 *Under assumption 2.2,*

i) If $0 < \gamma v_0 < \frac{8(2-\kappa_0)}{8-3\kappa_0}$*, then* $\bar{m} < m_{R3|R4}$.

ii) If $\frac{8(2-\kappa_0)}{8-3\kappa_0} < \gamma v_0 < \frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}}$*, then* $m_{R3|R4} < \bar{m} < m_{R4|R1}$

ii) If $\frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}} < \gamma v_0$*, then* $m_{R4|R1} < \bar{m}$

Proof. To determine whether $\bar{m}$ lies within $[0, m_{R3|R4}]$, $[m_{R3|R4}, m_{R4|R1}]$ or $[m_{R4|R1}, \infty]$ first notice that $\frac{v_0 S(m_0)}{(v_0-m_0)}$ is increasing in m_0. Therefore, the conditions on the parameters for each of these cases to hold are:

For $\bar{m} < m_{R3|R4}$ This is the case if $\frac{v_0 S(m_{R3|R4})}{\kappa_0(v_0-m_{R3|R4})} > \frac{4}{\kappa_0^2} \iff v_0\gamma < \frac{8(2-\kappa_0)}{8-3\kappa_0}$

For $m_{R3|R4} < \bar{m} < m_{R4|R1}$ From above $m_{R3|R4} < \bar{m} \iff \frac{8(2-\kappa_0)}{8-3\kappa_0} < v_0\gamma$

Now we need to unveil the condition for $\frac{v_0 S(m_{R4|R1})}{\kappa_0(v_0-m_{R4|R1})} > \frac{4}{\kappa_0^2} \iff v_0\gamma < \frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}} = \frac{4\rho(\rho-\kappa_0(\rho-1))}{(\rho-1)(4\rho-\kappa_0(\rho+1))}$. Therefore, this case occurs when

$$\frac{8(2-\kappa_0)}{8-3\kappa_0} < v_0\gamma < \frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}}$$

For $m_{R4|R1} < \bar{m}$ It follows directly from before

$$\frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}} < v_0\gamma$$

■

Lemma 2 provides us conditions on the parameters that fully describe the regimes in period 0 and period 1. The optimal m_0 therefore can be computed by simply computing the m_0 that maximizes EU in period 0 in each of the three cases in Lemma 2. For the proof of part 1 in Proposition **3** we only need to find a cutoff such that the polity stays in $R3$.We propose $\tau_L \equiv \frac{8(2-\kappa_0)}{8-3\kappa_0}$.

In this case, $v_0\gamma < \frac{8(2-\kappa_0)}{8-3\kappa_0}$ is equivalent to a regime described by $\bar{m} < m_{R3|R4}$.Note that $\frac{8(2-\kappa_0)}{8-3\kappa_0}$ is strictly decreasing in κ_0 so its maximum value is at $\kappa_0 = 1$. In this case $\frac{8(2-1)}{8-3\times 1} = \frac{8}{5} < 2$. Let us analyze the expected utility by segments:

Segment $[0, \bar{m}]$ Expected utility in period $t = 0$ is

$EU = v_0 - m_0 + \frac{\kappa_0}{4} v_0 S(m_0) = v_0 - m_0 + \frac{\kappa_0}{4} v_0 \left(1 + \frac{\kappa_0 + \gamma m_0}{4}\right) = v_0(1 + \frac{\kappa_0}{4} + \left(\frac{\kappa_0}{4}\right)^2) + m_0\left(\frac{\kappa_0 v_0 \gamma}{16} - 1\right)$

Since $\bar{m} < m_{R3|R4} \iff v_0\gamma < \frac{8(2-\kappa_0)}{8-3\kappa_0}$ then $\frac{\kappa_0 v_0 \gamma}{16} - 1 < 0$, To see why, replace $v_0\gamma = \frac{8(2-\kappa_0)}{8-3\kappa_0}$ in $\frac{\kappa_0 v_0\gamma}{16}$ so $\frac{\kappa_0 v_0 \gamma}{16} = \frac{\kappa_0\left(\frac{8(2-\kappa_0)}{8-3\kappa_0}\right)}{16} \leq \frac{\kappa_0 \frac{8}{5}}{16} < 1$.So the optimal choice is $m_0 = 0$.

Segment $[\bar{m}, m_{R3|R4}]$ Expected utility in period $t = 0$ is

$EU =$

$\frac{1}{2}\frac{1}{\sqrt{\kappa_0 v_0 (v_0 - m_0 + \frac{1}{4}(v_0\kappa_0 - m_0\kappa_0 + v_0\gamma m_0 - m_0^2\gamma))}}\left(\kappa_0 v_0\left(-1 - \frac{1}{4}\kappa_0 + \frac{1}{4}v_0\gamma - \frac{1}{4}2m_0\gamma\right)\right)$

$\frac{dEU}{dm} < 0 \iff -1 - \frac{1}{4}\kappa_0 + \frac{1}{4}v_0\gamma - \frac{1}{4}2m_0\gamma < 0 \iff v_0\gamma < 4\left(\frac{1}{4}2m_0\gamma + 1 + \frac{1}{4}\kappa_0\right)$.

If $4\left(\frac{1}{4}2m_0\gamma + 1 + \frac{1}{4}\kappa_0\right)$ is higher than $\frac{8(2-\kappa_0)}{8-3\kappa_0}$, our condition to be in this scenario $\bar{m} < m_{R3|R4}\left(\iff v_0\gamma < \frac{8(2-\kappa_0)}{8-3\kappa_0}\right)$ is a suficient condition for EU in this segment to be decreasing. So, it is direct to show that $4\left(\frac{1}{4}2m_0\gamma + 1 + \frac{1}{4}\kappa_0\right) > \frac{8(2-\kappa_0)}{8-3\kappa_0}$ because the right hand side is decreasing in κ_0, so its maximum is attained at $\kappa_0 = 1$ and it is equal to 8/5 which is smaller than any feasible value of the expression in the left hand side. In this segment, the utility is maximized at $m_0 = \bar{m}$, which is smaller than the utility at $m_0 = 0$ from the analysis in the case above.

$EU = \sqrt{\kappa_0 (v_0 - m_0) v_0 S(m_0)} = \sqrt{4(v_0 - \bar{m})^2} = 2(v_0 - \bar{m})$

Segment $[m_{R3|R4}, m_{R4|R1}]$ Expected utility in period $t = 0$ is

$EU = \sqrt{\kappa_0 (v_0 - m_0) v_0 \left(2 - \frac{1}{\kappa_0 + \gamma m_0}\right)}$

computing the first derivative with respecto m_0

$\frac{dEU}{dm_0} = \frac{\sqrt{\kappa_0 v_0}}{2}\frac{1}{\sqrt{(v_0 - m_0)\left(2 - \frac{1}{\kappa_0 + \gamma m_0}\right)}}\left(\frac{v_0\gamma}{(\kappa_0 + \gamma m_0)^2} - 2 + \left(\frac{1}{\kappa_0 + \gamma m_0} - \frac{m_0\gamma}{(\kappa_0 + \gamma m_0)^2}\right)\right)$ this quantity is less than zero, so the optimum is at $m_0 = m_{R3|R4}$, and this is smaller than the value of EU at $m_0 = 0$. To see why note that $\left(\frac{v_0\gamma}{(\kappa_0 + \gamma m_0)^2} - 2 + \left(\frac{1}{\kappa_0 + \gamma m_0} - \frac{m_0\gamma}{(\kappa_0 + \gamma m_0)^2}\right)\right) < 0$ is equivalent to $v_0\gamma < 2(\kappa_0 + \gamma m_0)^2 + m_0\gamma - (\kappa_0 + \gamma m_0) \equiv W$ after re-arranging terms. The right-hand side of this expression, W, is higher than $\tau_L = \frac{8(2-\kappa_0)}{8-3\kappa_0}$: It is increasing in m_0 so the smallest possible value is at $m_0 = m_{R3|R4}$. At this value the expression W is $8 - \kappa_0$, comparing

$$\frac{8(2-\kappa_0)}{8-3\kappa_0} < 8 - \kappa_0 \iff 0 < 48 - 24\kappa_0 + 3\kappa_0^2$$

which always hold in our case, because $\kappa_0 < 2$.

Segment $[m_{R4|R1}, \infty]$ $EU = \sqrt{\kappa_0 (v_0 - m_0) v_0 \frac{(\kappa_0 + \gamma m_0)(1+\rho)}{\kappa_0 + \gamma m_0 + \rho}}$

$\frac{dEU}{dm_0} =$

$\sqrt{\kappa_0 v_0}\frac{1}{2}\frac{1}{\sqrt{(v_0 - m_0)\frac{(\kappa_0 + \gamma m_0)(1+\rho)}{\kappa_0 + \gamma m_0 + \rho}}}\left(\frac{-(\kappa_0 + \gamma m_0)^2(1+\rho) - (\kappa_0 + \gamma m_0)\rho(1+\rho) - \gamma m_0\rho(1+\rho) + \gamma v_0\rho(1+\rho)}{(\kappa_0 + \gamma m_0 + \rho)^2}\right)$

This marginal EU is decreasing in m_0. To see why this is true, note that $-\left(\kappa_0+\gamma m_0\right)^2(1+\rho)- \left(\kappa_0+\gamma m_0\right)\rho(1+\rho)-\gamma m_0\rho(1+\rho)+\gamma v_0\rho(1+\rho)<0$ is equivalent to

$$v_0\gamma<\frac{1}{\rho}\left(\kappa_0+\gamma m_0\right)^2+\left(\kappa_0+\gamma m_0\right)+\gamma m_0.$$

The right hand side of this expression is increasing in m_0, so the minimum is attained at $m_0=m_{R4|R1}$ and it equals $\frac{\rho}{(\rho-1)^2}+2\frac{\rho}{(\rho-1)}-\kappa_0$. The highest possible value of γv_0, $\tau_L=\frac{8(2-\kappa_0)}{8-3\kappa_0}$ is smaller than $\frac{8}{5}$ which, in turn, is always smaller than $\frac{\rho}{(\rho-1)^2}+2\frac{\rho}{(\rho-1)}-\kappa_0$.

Therefore the maximum of EU in this segment is attained at $m_0=m_{R4|R1}$.

In sum, the global maximum in this case is $m_0=0$. This follows from the fact that EU is continuous: $S()$ is continuous for all m_0 and EU in period $t=0$ is also continuous at $\bar{m}$: in $t=0$ in segment $[0,\bar{m}]$, EU evaluated at $\bar{m}$ is $2(v_0-\bar{m})$ which is equal to the EU in segment $[\bar{m},m_{R3|R4}]$ evaluated at $\bar{m}$. This can be shown noticing that $\frac{\kappa_0 v_0 S(\bar{m})}{4}=v_0-\bar{m}$, and reeplacing in EU in segment $[0,\bar{m}]$. Thus, the polity will stay at R3 in period 1.

For the proof of part 2. and 3. of Proposition **3** (the existence of cutoffs such that the polity will move away from the trap represented by region R3 in period 1), we consider the case in which $\frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}}<v_0\gamma$. In this case $m_{R4|R1}<\bar{m}$ by Lemma 2. We proceed by analyzing the optimal decision of m_0 under different segments:

Segment $[0,m_{R3|R4}]$ Expected utility in period $t=0$ is

$EU=v_0-m_0+\frac{\kappa_0}{4}v_0S(m_0)=v_0-m_0+\frac{\kappa_0}{4}v_0\left(1+\frac{\kappa_0+\gamma m_0}{4}\right)=v_0(1+\frac{\kappa_0}{4}+\left(\frac{\kappa_0}{4}\right)^2)+m_0\left(\frac{\kappa_0 v_0\gamma}{16}-1\right)$

In this case, it is not always the case that $\frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}}>\frac{16}{\kappa_0}$ under Assumption 2.2. In general, if γv_0 is higher than $\tau_M\equiv\max\left\{\frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}},\frac{16}{\kappa_0}\right\}$ then the polity will move away from the trap region R3. Next, we show that it may either stay in R4 (peace) or move to R1 (peace and prosperity).

Segment $[m_{R3|R4},m_{R4|R1}]$ Expected utility in period $t=0$ is $EU=v_0-m_0+\frac{\kappa_0}{4}v_0S(m_0)=v_0-m_0+\frac{\kappa_0}{4}v_0\left(2-\frac{1}{\kappa_0+\gamma m_0}\right)$

The marginal utility of m_0 is: $-1+\frac{\kappa_0 v_0}{4}\frac{\gamma}{(\kappa_0+\gamma m_0)^2}$. The value of m_0 that maximizes the EU is $m_0=\frac{1}{\gamma}\left(\sqrt{\frac{\kappa_0 v_0\gamma}{4}}-\kappa_0\right)$. For the optimum to be interior we need to compare it with the boundaries of this region $m_{R3|R4}$ and $m_{R4|R1}$. This is the case when $16/\kappa_0<\gamma v_0<\left(\frac{\rho}{\rho-1}\right)^2\frac{4}{\kappa_0}$ then the optimum is $m_0=\frac{1}{\gamma}\left(\sqrt{\frac{\kappa_0 v_0\gamma}{4}}-\kappa_0\right)$.[13] Thus, if $\tau_M<\gamma v_0<$

[13] The slope in segment $[0,m_{R3|R4}]$ is positive, this plus continuity makes $\frac{1}{\gamma}\left(\sqrt{\frac{c_0 v_0\gamma}{4}}-c_0\right)$ the optimum.

$\tau_H \equiv \max\left\{ \frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}}, \left(\frac{\rho}{\rho-1}\right)^2 \frac{4}{\kappa_0} \right\}$ then the polity reaches R4. If $\tau_H < \gamma v_0$ then the polity conquers peace and prosperity in R1. Now we explore if it is possible to move to R1.

Segment $[m_{R4|R1}, \bar{m}]$ $\quad EU = v_0 - m_0 + \frac{\kappa_0}{4} v_0 S(m_0) = v_0 - m_0 + \frac{\kappa_0}{4} v_0 \left(\frac{(\kappa_0+\gamma m_0)(1+\rho)}{\kappa_0+\gamma m_0+\rho}\right)$

$\frac{dEU}{dm_0} = \frac{\kappa_0 v_0}{4} \frac{(\kappa_0+\gamma m_0)\gamma(1+\rho)+\gamma\rho(1+\rho)-(\kappa_0+\gamma m_0)\gamma(1+\rho)}{(\kappa_0+\gamma m_0+\rho)^2} - 1 = \frac{\kappa_0 v_0}{4} \frac{\gamma\rho(1+\rho)}{(\kappa_0+\gamma m_0+\rho)^2} - 1$ this implies the optimum $m_0 = \frac{1}{\gamma}\left(\sqrt{\frac{\kappa_0 v_0 \gamma}{4}\rho(1+\rho)} - \kappa_0 - \rho\right)$. It is streightforward to show that if $\tau_H = \max\left\{ \frac{\frac{4}{\kappa_0}\left(\frac{\rho}{\rho-1}-\kappa_0\right)}{\frac{4}{\kappa_0}-\frac{(1+\rho)}{\rho}}, \left(\frac{\rho}{\rho-1}\right)^2 \frac{4}{\kappa_0} \right\} < \gamma v_0$ then the optimum is interior: $m_0 = \frac{1}{\gamma}\left(\sqrt{\frac{\kappa_0 v_0 \gamma}{4}\rho(1+\rho)} - \kappa_0 - \rho\right) > m_{R4|R1}$. This implies that for $\gamma v_0 > \tau_H$ the polity will be in the interior of R1 in period 1, reaching growth and prosperity.■

Proof of Proposition 4: As in the proof of Proposition 3, let us call $\bar{m}$ the value of m_0 that satisfies the equation: $\frac{v_0 S(\bar{m})}{\kappa_0(v_0-\bar{m})} = \frac{4}{\kappa_0^2}$. $\bar{m}$ is the value of m_0 at which regimes change in period $t = 0$.

Let us call $m_{R3|R2} = \frac{1}{\gamma}\left(\frac{4}{\rho} - \kappa_0\right)$ and $m_{R2|R1} = \frac{1}{\gamma}(\rho - \kappa_0)$ the values in which regimes change in period $t = 1$. The following Lemma shows the conditions on the parameters such that for any given m_0 we can fully describe the EU in period $t = 0$.

Lemma 3 *Under assumption 2.2,*

i) If $0 < \gamma v_0 < \frac{16-4\kappa_0\rho}{4\rho-(1+\rho)\kappa_0}$ *then* $\bar{m} < m_{R3|R2}$

ii) If $\frac{16-4\kappa_0\rho}{4\rho-(1+\rho)\kappa_0} < \gamma v_0 < \frac{8(\rho-\kappa_0)}{8-(1+\rho)\kappa_0}$ *then* $m_{R3|R2} < \bar{m} < m_{R2|R1}$

ii) If $\frac{8(\rho-\kappa_0)}{8-(1+\rho)\kappa_0} < \gamma v_0$ *then* $m_{R2|R1} < \bar{m}$

Proof. It follows from replacing the definitions of $\bar{m}, m_{R3|R2}, m_{R2|R1}$ and following steps analogous to Lemma 2.■

For the proof of part 1 in Proposition 4 we only need to find a cutoff such that the polity stays in $R3$. We propose $\sigma_L \equiv \frac{16-4\kappa_0\rho}{4\rho-(1+\rho)\kappa_0}$.

In this case, $v_0\gamma < \sigma_L = \frac{16-4\kappa_0\rho}{4\rho-(1+\rho)\kappa_0}$ is equivalent to a regime in which $\bar{m} < m_{R3|R2}$. Notice that $\frac{16-4\kappa_0\rho}{4\rho-(1+\rho)\kappa_0}$ is decreasing in both ρ and κ_0. Thus, the highest feasible value of this expression is attained at $\rho = 2$ and $\kappa_0 = 1$, $\frac{16-4\times1\times2}{4\times2-(1+2)\times1} = \frac{8}{5}$. Let us analyze the expected utility by segments:

Segment $[0, \bar{m}]$ Expected utility in period $t = 0$ is

$EU = v_0 - m_0 + \frac{\kappa_0}{4} v_0 S(m_0) = v_0 - m_0 + \frac{\kappa_0}{4} v_0 \left(1 + \frac{\kappa_0+\gamma m_0}{4}\right) = v_0(1 + \frac{\kappa_0}{4} + \left(\frac{\kappa_0}{4}\right)^2) + m_0\left(\frac{\kappa_0 v_0 \gamma}{16} - 1\right)$

Since $\bar{m} < m_{R3|R4} \iff v_0\gamma < \sigma_L$ then $\frac{\kappa_0 v_0 \gamma}{16} - 1 < 0$ so the optimal choice is $m_0 = 0$.[14]

Segment $[\bar{m}, m_{R3|R2}]$ Expected utility in period $t = 0$ is

$EU = \sqrt{\kappa_0 v_0 (v_0 - m_0 + \frac{1}{4}(v_0 - m_0)(\kappa_0 + \gamma m_0))}$

$\frac{dEU}{dm_0} = \frac{1}{2}\frac{1}{\sqrt{\kappa_0 v_0 (v_0 - m_0 + \frac{1}{4}(v_0\kappa_0 - m_0\kappa_0 + v_0\gamma m_0 - m_0^2\gamma))}}\left(\kappa_0 v_0\left(-1 - \frac{1}{4}\kappa_0 + \frac{1}{4}v_0\gamma - \frac{1}{4}2m_0\gamma\right)\right)$

$\frac{dEU}{dm} < 0 \iff -1 - \frac{1}{4}\kappa_0 + \frac{1}{4}v_0\gamma - \frac{1}{4}2m_0\gamma < 0 \iff v_0\gamma < 4\left(\frac{1}{4}2m_0\gamma + 1 + \frac{1}{4}\kappa_0\right)$.

$4\left(\frac{1}{4}2m_0\gamma + 1 + \frac{1}{4}\kappa_0\right)$ is higher than $\frac{16 - 4\kappa_0\rho}{4\rho - (1+\rho)\kappa_0}$. Thus, EU in this segment is decreasing. It is straightforward to show that $4\left(\frac{1}{4}2m_0\gamma + 1 + \frac{1}{4}\kappa_0\right) > \frac{16 - 4\kappa_0\rho}{4\rho - (1+\rho)\kappa_0}$: The lowest possible value of the left-hand side is at $m_0 = 0$, $4 + \kappa_0$. If we compare

$$4 + \kappa_0 > \frac{16 - 4\kappa_0\rho}{4\rho - (1+\rho)\kappa_0}$$

is equivalent to

$$16 + 4\kappa_0 + \kappa_0^2 + \kappa_0\rho\kappa_0 < 16\rho + 4\kappa_0\rho$$

the left-hand side of this expression can be at most 36, while the right-hand side one, at least 40 under Assumption 2.2. As a result, the maximum is attained at $m_0 = 0$ in the interval $[0, m_{R3|R2}]$.

Segment $[m_{R3|R2}, m_{R2|R1}]$ Expected utility in period $t = 0$ is

$EU = \sqrt{\kappa_0 (v_0 - m_0) v_0 \left(\sqrt{\frac{\kappa_0 + \gamma m_0}{\rho}}\frac{(1+\rho)}{2}\right)}$ computing the first derivative with respect to m_0

$\frac{dEU}{dm_0} = \frac{\sqrt{\kappa_0 v_0 \frac{(1+\rho)}{2}}}{2}\frac{1}{\sqrt{(v_0 - m_0)\left(\sqrt{\frac{\kappa_0 + \gamma m_0}{\rho}}\right)}}\left(-\sqrt{\frac{\kappa_0 + \gamma m_0}{\rho}} + (v_0 - m_0)\frac{1}{2}\left(\frac{\kappa_0 + \gamma m_0}{\rho}\right)^{-1/2}\frac{\gamma}{\rho}\right)$ this quantity is less than zero, so the optimum is at $m_0 = m_{R3|R4}$, and this is smaller than the value of EU at $m_0 = 0$. To see why, $\left(-\sqrt{\frac{\kappa_0 + \gamma m_0}{\rho}} + (v_0 - m_0)\frac{1}{2}\left(\frac{\kappa_0 + \gamma m_0}{\rho}\right)^{-1/2}\frac{\gamma}{\rho}\right) < 0$ is equivalent to $v_0\gamma < 2(\kappa_0 + \gamma m_0) + m_0\gamma$ after re-arranging terms. The right-hand side of this expression is higher than $\frac{16 - 4\kappa_0\rho}{4\rho - (1+\rho)\kappa_0}$ for all m_0: It is increasing in m_0 so the smallest possible value is at $m_0 = 0$. At this value the minimal value of the expression expression is $2\kappa_0$, comparing

$$\frac{16 - 4\kappa_0\rho}{4\rho - (1+\rho)\kappa_0} < 2\kappa_0$$

which holds in our case since $\frac{16 - 4\kappa_0\rho}{4\rho - (1+\rho)\kappa_0} < \frac{8}{5} < 2$, because $1 \le \kappa_0 < 2$.

[14] Just replace $v_0\gamma = \frac{16 - 4c_0\beta}{4\beta - (1+\beta)c_0}$ in $\frac{c_0 v_0 \gamma}{16}$ so $\frac{c_0 v_0 \gamma}{16} = \frac{c_0\left(\frac{16 - 4c_0\beta}{4\beta - (1+\beta)c_0}\right)}{16}$ which is smaller than $\frac{c_0\left(\frac{8}{5}\right)}{16}$ which in turn is less than 1.

Segment $[m_{R2|R1}, \infty]$ $\quad EU = \sqrt{\kappa_0 (v_0 - m_0) v_0 \frac{(\kappa_0+\gamma m_0)(1+\rho)}{\kappa_0+\gamma m_0+\rho}}$

$\frac{dEU}{dm_0} = \sqrt{\kappa_0 v_0} \frac{1}{2} \frac{1}{\sqrt{(v_0-m_0)\frac{(\kappa_0+\gamma m_0)(1+\rho)}{\kappa_0+\gamma m_0+\rho}}} \left(\frac{-(\kappa_0+\gamma m_0)^2(1+\rho)-(\kappa_0+\gamma m_0)\rho(1+\rho)-\gamma m_0\rho(1+\rho)+\gamma v_0\rho(1+\rho)}{(\kappa_0+\gamma m_0+\rho)^2} \right)$

This marginal EU is decreasing in m_0. To see why, $-(\kappa_0 + \gamma m_0)^2 (1+\rho) - (\kappa_0 + \gamma m_0)\rho(1+\rho) - \gamma m_0 \rho(1+\rho) + \gamma v_0 \rho(1+\rho) < 0$ is equivalent to

$$v_0 \gamma < \frac{1}{\rho} (\kappa_0 + \gamma m_0)^2 + (\kappa_0 + \gamma m_0) + \gamma m_0.$$

The right hand side of this expression is increasing in m_0, so the minimum of this expression is attained at $m_0 = m_{R2|R1}$ and it equals $3\rho - \kappa_0$. Comparing the highest possible value of γv_0, $\frac{16-4\kappa_0\rho}{4\rho-(1+\rho)\kappa_0}$ with the smallest value $3\rho - \kappa_0$ the result follows.

In sum, the global maximum when $\bar{m} < m_{R3|R2} \left(\iff v_0\gamma < \frac{16-4\kappa_0\rho}{4\rho-(1+\rho)\kappa_0} = \sigma_L \right)$ **is** $m_0 = 0$. This follows from the fact that EU is continuous: $S()$ is continuous for all m_0 and EU in period $t = 0$ is also continuous at $\bar{m}$: in $t = 0$ in segment $[0, \bar{m}]$, EU evaluated at $\bar{m}$ is $2(v_0 - \bar{m})$ which is equal to the EU in segment $[\bar{m}, m_{R3|R4}]$ evaluated at $\bar{m}$. This can be shown noticing that $\frac{\kappa_0 v_0 S(\bar{m})}{4} = v_0 - \bar{m}$, and reeplacing in EU in segment $[0, \bar{m}]$. Thus, the polity will stay at R3 in period 1.

For the proof of part 2. and 3. of Proposition **??** (the existence of cutoffs such that the polity will move away from the trap represented by region R3 in period 1), we consider the case in which $\frac{8(\rho-\kappa_0)}{8-(1+\rho)\kappa_0} < v_0\gamma$. In this case $m_{R2|R1} < \bar{m}$ by Lemma 3. We proceed by analyzing the optimal decision of m_0 under different segments:

Segment $[0, m_{R3|R2}]$ Expected utility in period $t = 0$ is

$EU = v_0 - m_0 + \frac{\kappa_0}{4} v_0 S(m_0) = v_0 - m_0 + \frac{\kappa_0}{4} v_0 \left(1 + \frac{\kappa_0+\gamma m_0}{4}\right) = v_0(1 + \frac{\kappa_0}{4} + \left(\frac{\kappa_0}{4}\right)^2) + m_0 \left(\frac{\kappa_0 v_0 \gamma}{16} - 1\right)$

In this case, $\frac{8(\rho-\kappa_0)}{8-(1+\rho)\kappa_0} < \frac{16}{\kappa_0}$ under Assumption 2.2. Thus, there exists a threshold $\frac{16}{\kappa_0}$ such that $\frac{8(\rho-\kappa_0)}{8-(1+\rho)\kappa_0} < v_0\gamma < \frac{16}{\kappa_0}$, $m_0 = 0$ in this segment. If $\gamma v_0 > \frac{16}{\kappa_0}$ then $m_0 = m_{R3|R2}$.

At $m_0 = 0$ the EU in this segment is$= v_0 - 0 + \frac{\kappa_0}{4} v_0 \left(1 + \frac{\kappa_0+0}{4}\right) = v_0 + \frac{\kappa_0 v_0}{4} + \frac{\kappa_0^2}{16} v_0$

Segment $[m_{R3|R2}, m_{R2|R1}]$ $\quad EU = v_0 - m_0 + \frac{\kappa_0}{4} v_0 \left(\sqrt{\frac{\kappa_0+\gamma m_0}{\rho}} \frac{(1+\rho)}{2} \right)$

The marginal utility of m_0 is: $-1 + \frac{\kappa_0 v_0}{4} \frac{(1+\rho)}{2} \frac{1}{2} \sqrt{\frac{\rho}{\kappa_0+\gamma m_0}} \frac{\gamma}{\rho}$. This marginal utility may be either negative, zero or positive for m_0 in the segment $[m_{R3|R2}, m_{R2|R1}]$. It is negative in the segment $[m_{R3|R2}, m_{R2|R1}]$ if and only if the value at which the marginal utility is zero, $m_0 = \frac{\left(\frac{\kappa_0 v_0 \gamma}{16} \frac{(1+\rho)\sqrt{\rho}}{\rho}\right)^2 - \kappa_0}{\gamma}$, is smaller than $m_{R3|R2}$:

$$m_0 = \frac{\left(\frac{\kappa_0 v_0 \gamma}{16}\frac{(1+\rho)\sqrt{\rho}}{\rho}\right)^2 - \kappa_0}{\gamma} < m_{R3|R2} = \frac{(\frac{4}{\rho} - \kappa_0)}{\gamma}$$
$$\Longleftrightarrow v_0\gamma < \left(\frac{16}{\kappa_0}\right)\frac{2}{1+\rho}$$

The marginal utility is zero–so the solution is interior in $R2$–if $m_0 = \frac{\left(\frac{\kappa_0 v_0 \gamma}{16}\frac{(1+\rho)\sqrt{\rho}}{\rho}\right)^2 - \kappa_0}{\gamma}$, is smaller than $m_{R2|R1}$ (and higher than $m_{R3|R2}$) or if

$$m_{R3|R2} < m_0 = \frac{\left(\frac{\kappa_0 v_0 \gamma}{16}\frac{(1+\rho)\sqrt{\rho}}{\rho}\right)^2 - \kappa_0}{\gamma} < m_{R3|R2} = \frac{(\rho - \kappa_0)}{\gamma}$$

$$\Longleftrightarrow \left(\frac{16}{\kappa_0}\right)\frac{2}{1+\rho} < v_0\gamma < \left(\frac{16}{\kappa_0}\right)\frac{\rho}{1+\rho}$$

Finally, the marginal utility is positive in this segment $[m_{R3|R2}, m_{R2|R1}]$ if $\left(\frac{16}{\kappa_0}\right)\frac{\rho}{1+\rho} < v_0\gamma$.

Notice that a sufficient condition to have an interior maximum in $R2$ is that the EU evaluated at $m_{R2|R1}$, $v_0 - \frac{\rho-\kappa_0}{\gamma} + \frac{\kappa_0}{4}v_0\frac{(1+\rho)}{2}$, is higher than the EU evaluated at $m_0 = 0$, $v_0 + \frac{\kappa_0 v_0}{4} + \frac{\kappa_0^2}{16}v_0$, when $\left(\frac{16}{\kappa_0}\right)\frac{2}{1+\rho} < v_0\gamma < \left(\frac{16}{\kappa_0}\right)\frac{\rho}{1+\rho}$. This sufficient condition is

$$v_0 - \frac{\rho - \kappa_0}{\gamma} + \frac{\kappa_0}{4}v_0\frac{(1+\rho)}{2} > v_0 + \frac{\kappa_0 v_0}{4} + \frac{\kappa_0^2}{16}v_0$$
$$\frac{\gamma v_0 \kappa_0}{16}(\rho - \kappa_0 + \rho - 2) > \rho - \kappa_0$$
$$\gamma v_0 > \left(\frac{16}{\kappa_0}\right)\frac{\rho - \kappa_0}{\rho - \kappa_0 + \rho - 2}.$$

Notice that $\left(\frac{16}{\kappa_0}\right)\frac{\rho-\kappa_0}{\rho-\kappa_0+\rho-2}$ could be either below $\left(\frac{16}{\kappa_0}\right)\frac{2}{1+\rho}$, above $\left(\frac{16}{\kappa_0}\right)\frac{\rho}{1+\rho}$ or in between them. Let us then define $\sigma_M \equiv \max\left\{\left(\frac{16}{\kappa_0}\right)\frac{\rho-\kappa_0}{\rho-\kappa_0+\rho-2}, \left(\frac{16}{\kappa_0}\right)\frac{2}{1+\rho}\right\}$ and $\sigma_H = \max\left\{\left(\frac{16}{\kappa_0}\right)\frac{\rho}{1+\rho}, \left(\frac{16}{\kappa_0}\right)\frac{\rho-\kappa_0}{\rho-\kappa_0+\rho-2}\right\}$. Hence, if $\sigma_M < v_0\gamma < \sigma_H$ then the polity stays in $R2$. If $\sigma_H < v_0\gamma$ then the polity moves to $R1$. The following shows the polity actually moves to the interior of $R1$ in this case.

Segment $[m_{R4|R1}, \bar{m}]$ $\quad EU = v_0 - m_0 + \frac{\kappa_0}{4}v_0 S(m_0) = v_0 - m_0 + \frac{\kappa_0}{4}v_0\left(\frac{(\kappa_0+\gamma m_0)(1+\rho)}{\kappa_0+\gamma m_0+\rho}\right)$

In this case, the marginal utility is $\frac{dEU}{dm_0} = \frac{\kappa_0 v_0}{4}\frac{(\kappa_0+\gamma m_0)\gamma(1+\rho)+\gamma\rho(1+\rho)-(\kappa_0+\gamma m_0)\gamma(1+\rho)}{(\kappa_0+\gamma m_0+\rho)^2} - 1 = \frac{\kappa_0 v_0}{4}\frac{\gamma\rho(1+\rho)}{(\kappa_0+\gamma m_0+\rho)^2} - 1$, so the interior optimum is $m_0 = \frac{1}{\gamma}\left(\sqrt{\frac{\kappa_0 v_0 \gamma}{4}}\rho(1+\rho) - \kappa_0 - \rho\right)$. It is stratightforward (using arguments analogous to the case in segment $[m_{R3|R2}, m_{R2|R1}]$) to show that if $\frac{16}{\kappa_0}\frac{\rho}{(1+\rho)} < \gamma v_0$ then $m_0 = \frac{1}{\gamma}\left(\sqrt{\frac{\kappa_0 v_0 \gamma}{4}\rho(1+\rho)} - \kappa_0 - \rho\right) > m_{R2|R1}$. This implies that for $\gamma v_0 > \max\left\{\frac{16}{\kappa_0}\frac{\rho}{(1+\rho)}, \left(\frac{16}{\kappa_0}\right)\frac{\rho-\kappa_0}{\rho-\kappa_0+\rho-2}\right\} = \sigma_H$ the polity moves to the interior of R1 in period 1, and the it conquers growth and order. ■

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